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A STRUCTURAL THEORY FOR THE PERCEPTION OF MORSE CODE SIGNALS
AND RELATED RHYTHMIC PATTERNS.

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MORSE CODE,

THE PRIMARY PURPOSE OF THIS DISSERTATION IS TO DEVELOP A
STRUCTURAL THEORY, ALONG FACET-THEORETIC LINES, FOR THE
PERCEPTION OF MORSE CODE SIGNALS AND RELATED RHYTHMIC
PATTERNS. AS STEPS IN THE DEVELOPMENT OF THIS THEORY, MODELS
FOR TWO SETS OF SIGNALS ARE PROPOSED AND TESTED. THE FIRST
MODEL IS FOR A SET COMPRISED OF ALL SIGNALS OF THE
INTERNATIONAL MORSE CODE (IMC) IN ADDITION TO SOME SIMILAR
DOT AND DASH PATTERNS. THE SECOND MODEL IS FOR A SET
COMPRISED OF 32 MORSE-LIKE RHYTHMIC PATTERNS IN WHICH THE
SILENT INTERVALS BETWEEN THE DOTS AND DASHES CAN BE EITHER
SHORT OR LONG. AN EXPERIMENT BY ROTHKOPF SUPPLIES THE DATA
USED TO TEST THE FIRST MODEL. AN EXPERIMENT CARRIED OUT BY
THE PRESENT INVESTIGATOR PROVIDED THE DATA USED TO TEST THE
SECOND MODEL. IN ROTHKOPF'S STUDY 598 AIRMEN (WHO HAD HAD NO
PREVIOUS MORSE CODE TRAINING) AWAITING BASIC TRAINING WERE
PRESENTED PAIRS OF IMC SIGNALS AND WERE INSTRUCTED TO RESPOND
"SAME" OR "DIFFERENT" ON AN IBM ANSWER SHEET TO EACH OF THESE
PAIRS ACCORDING TO WHETHER THEY PERCEIVED THE TWO SIGNALS OF
THE PAIR TO BE THE SAME OR DIFFERENT. IN THE PRESENT
INVESTIGATOR'S EXPERIMENT, 324 FEMALE UNDERGRADUATES AT THE
UNIVERSITY OF MICHIGAN ENROLLED IN INTRODUCTORY PSYCHOLOGY
COURSES WERE PRESENTED PAIRS OF THE MORSE-LIKE RHYTHMIC
PATTERNS, RATHER THAN PAIRS OF IMC SIGNALS, AND WERE
INSTRUCTED TO INDICATE WHETHER THEY PERCEIVED THE SIGNALS OF
EACH OF THESE PAIRS AS SAME OR DIFFERENT. IN TESTING BOTH
MODELS THE PERCENTAGE OF "SAME" RESPONSES TO EACH ORDERED
PAIR OF SIGNALS IS USED AS A MEASURE OF THE CONFUSABILITY OF
THE SIGNALS OF THE ORDERED PAIR. USING THIS CONFUSABILITY
MEASURE, THE FIRST AND SECOND MODELS ARE SHOWN TO BE
OVERWHELMINGLY SUPPORTED BY THE DATA. OTHER PATTERNS OF
SIGNAL CONFUSION IN THE TWO EXPERIMENTS ARE ALSO DESCRIBED.
THIS DISSERTATION APPEARS IN "STUDIES IN LANGUAGE AND
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ABSTRACT

A STRUCTURAL THEORY FOR THE PERCEPTION
OF MORSE CODE SIGNALS AND
RELATED RHYTHMIC PATTERNS

By Myron Wish

The primary purpose of this dissertation is to develop a structural theory, along facet-theoretic lines, for the perception of Morse Code signals and related rhythmic patterns. As steps in the development of this theory, models for two sets of signals are proposed and tested. The first model is for a set comprised of all signals of the International Morse Code (IMC) in addition to some similar dot-and-dash patterns. The second model is for a set comprised of 32 Morse-like rhythmic patterns in which the silent intervals between the dots and dashes can be either short or long.

In both models, signal confusability is systematically related to structural interrelations among the signals. This is accomplished in each model by:

- (1) Defining an order relation among the signals;
- (2) Hypothesizing that for any three signals, x , y , z , if $x < y < z$ or $z < y < x$, then x will be confused more often with y than with z ;
- (3) Defining certain kinds of betweenness among the signals; and
- (4) Hypothesizing that for any three signals x , y , z , if y is between x and z in all of the proposed senses, then x will be confused more often with y than with z .

An experiment by Rothkopf supplies the data used to test the first model. An experiment carried out by the present investigator provides the data used to test the second model. In Rothkopf's study 598 airmen (who had had no previous Morse Code training) awaiting basic training were presented pairs of IMC signals and were instructed to respond "same" or "different" on an IBM answer sheet to each of these pairs according to whether they perceived the two signals of the pair to be the same or different. In the present investigator's experiment, 324 female undergraduates at the University of Michigan enrolled in introductory psychology courses were presented pairs of the Morse-like rhythmic patterns, rather than pairs of IMC signals, and were instructed to indicate whether they perceived the signals of each of these pairs as same or different.

In testing both models the percentage of "same" responses to each ordered pair of signals is used as a measure of the confusability of the signals of that ordered pair. Using this confusability measure, the first and second models are shown to be overwhelmingly supported by the data.

Other patterns of signal confusion in the two experiments are also described. In both experiments it is observed that (1) signals having the same number of components are confused more often if they differ on the final component than if they differ on any other single component, and (2) if $x < y$, and x has fewer dashes and/or fewer components than y , then x and y are judged to be "same" more often if x is presented before y than if y is presented before x .

In analyzing separately the responses made in the first, middle, and final thirds of the present investigator's experiment, it is discovered that although the number of "same" judgments decreases from the first to the middle to the final portion of the experimental sessions, the patterns of confusion remain quite constant.

After presenting both models and testing the data from both experiments, the models for both sets of signals are integrated into a single, more general structural theory. Finally, some implications of this thesis for future research are discussed.

CHAPTER 2. THE FIRST MODEL

I. The Universe of Content

Consider a set, Δ , whose elements are the two elementary components of Morse Code signals, dot (denoted ".") and dash (denoted "-"). The 2-component signals of the International Morse Code (IMC) -- (.), (.-), (-.), and (--) -- are the four elements of the product set, or Cartesian product, $\Delta \times \Delta$. Likewise, the eight 3-component signals of the IMC are the elements of the product set, $\Delta \times \Delta \times \Delta$. Let μ_1 be the set which is equal to the union of sets Δ , $\Delta \times \Delta$, $\Delta \times \Delta \times \Delta$, $\Delta \times \Delta \times \Delta \times \Delta$, and $\Delta \times \Delta \times \Delta \times \Delta \times \Delta$. The set of all signals in the IMC is a proper subset of μ_1 ; i.e., all IMC signals are elements of μ_1 , but not all elements of μ_1 are in the IMC.

As is the case for signals in the IMC, in all elements of μ_1 the duration of a dash is three times that of a dot, and the interval between adjacent components of a signal has the same duration as a dot. The duration of a dot will be the unit of temporal length for these signals. The temporal length of a signal, which is defined to be the total number of units in that signal, is equal in units to the number of dots in the signal, plus the number of intervals between adjacent components in the signal (one less than the number of components in the signal), plus three times the number of dashes in the signal.

In Table 1 all elements of μ_1 are listed along with their temporal lengths. The alphabetic or numerical equivalents of the IMC signals are also given in the table. The entire set, μ_1 , rather than just the 36 IMC signals will be the universe of content for the first model.

TABLE 1

The Signal Universe for Model 1

Signal	International Morse Code Equivalent	Temporal Length in Units	Signal	International Morse Code Equivalent	Temporal Length in Units
.	E	1	• • • • •	5	9
—	T	3	• • • • —	4	11
• •	I	3	• • • • •		11
• •	A	5	• • • • •		11
• •	N	5	• — • • •	6	11
— —	M	7	— • • • •	3	13
• • •	S	5	• • • — •		13
• • •	U	7	• • • • •		13
• • •	R	7	• — • • •		13
• • •	D	7	• — • • •		13
• — •	W	9	— • • • •		13
— • •	K	9	— • • • •		13
— • •	G	9	— • • • •		13
• • •	O	11	• • • • •	7	13
• • • •	H	7	• • • • •	2	15
• • • •	V	9	• • • • •		15
• • • •	F	9	• • • • •		15
• • • •	L	9	• • • • •		15
• • • •	B	9	• • • • •		15
• • • •		11	• • • • •		15
• • • •	P	11	• • • • •		15
• • • •	X	11	• • • • •	8	15
• • • •	C	11	• • • • •	1	17
• • • •	Z	11	• • • • •		17
• • • •	J	13	• • • • •		17
• • • •	Y	13	• • • • •		17
• • • •	Q	13	• • • • •	9	17
• • • •		13	• • • • •	0	19
• • • •		15			

II. The Structural Hypotheses

A. The First Hypothesis

An ordering of the signals in terms of temporal length is defined below. This definition takes into account the precise sequence of dots and dashes in the signals.

Definition 1.

For all $x, y \in \mu_1$, $x < y$ if and only if (iff) it is possible to transform x to y by applying to x one or both of the following operations:

- (1) changing at least one dot to a dash;
- (2) adding at least one component.

If x and y have the same number of components, then according to the definition, $x < y$ iff the duration of each component of x is less than or equal to the duration of the corresponding component of y , and the duration of at least one component of x is less than the duration of the corresponding component of y ; i.e., iff x is weakly dominated by y .

Examples:

1. Let $x = (..-)$ and $y = (---)$.

Since x can be transformed to y by changing its first and second components to dashes, $(..-) < (---)$.

2. Let $x = (--.)$ and $y = (---..)$.

Since x can be transformed to y by adding to x a dash at the beginning and a dot at the end (and in many other ways also), $(--.) < (---..)$.

3. Let $x = (-..)$ and $y = (.--.)$.

Although it is not possible to transform x to y by applying only one of the operations above, it is possible to do so by changing the second component of x to a dash (the first operation) and adding a dot at the beginning of x (the second operation). Therefore $(-..) < (.--.)$.

Since not all signals are comparable, e.g., neither $(..-)<(--.)$ nor $(--.)<(..-)$, and neither $(.-.)<(.---)$ nor $(.---)<(.-.)$, Definition 1 establishes only a partial order on μ_1 .

Definition 2.

For all $x, y \in \mu_1$, y is an immediate successor of x iff $x < y$, and there is no $z \in \mu_1$ such that $x < z < y$.

It follows from this definition that y is an immediate successor of x iff x can be transformed to y by either changing one of x 's dots to a dash or by adding one dot to x .

Definition 3.

For all $x, y, z \in \mu_1$ y is immediately bounded by x and z iff one of the following is true:

- (1) y is an immediate successor of x , and z is an immediate successor of y ; or
- (2) y is an immediate successor of z , and x is an immediate successor of y .

Thus, since $(...-)$ is an immediate successor of $(..-)$ and $(..-)$ is an immediate successor of $(...)$, $(..-)$ is immediately bounded by $(...)$ and $(...-)$, or equivalently, $(..-)$ is immediately bounded by $(...-)$ and $(...)$. Definitions 2 and 3 will be utilized at later points in this paper.

Definition 4.

For all $x, y, z \in \mu_1$, x is closer to y than to z in the ordering on μ_1 iff $x < y < z$ or $z < y < x$.

Due to the symmetry of this definition, x is closer to y than to z in the ordering on μ_1 iff z is closer to y than to x in the ordering on μ_1 . Since $(..-)<(.---)<(.----)$, $(..-)$ is closer to $(.---)$ than to $(.----)$ in the ordering on μ_1 , and $(.---)$ is closer to $(.----)$ than to

(..-) in the ordering on μ_1 . Having defined a notion of relative closeness for signals, we are now prepared to state the first hypothesis.

Hypothesis 1.

If x is closer to y than to z in the ordering on μ_1 , then x will be confused more often with y than with z .

For example, it is predicted that (..-) will be confused more often with (....-) than with (..---), and that (..---) will be confused more often with (....-) than with (..-).

B. The Second Hypothesis

In the first hypothesis a correspondence between relative closeness in the ordering on μ_1 and relative confusability was proposed. In the second hypothesis predictions of relative confusability are made on the basis of some other structural properties of the signals. For example, two of the predictions that follow from the second hypothesis are that (..-) will be confused more often with (.-) than with (--.), and that (--.) will be confused more often with (-..) than with (..-). An intuitive justification for these predictions is that whereas (.-) differs from (..-) only on the second component and differs from (--.) only in terms of an inversion of the first and third components, (.-) differs from (--.) in both respects. In general, if x differs from z in every respect that it differs from y and in some other respects as well, then x should be confused more often with y than with z .

I shall now proceed to give a more formal basis for the hypothesis of which this prediction is a special case.

Definition 5.

For all $x, y, z \in \mu_1$, y is component-wise between x and z iff:

- (1) x, y and z have the same number of components, and
- (2) for all i , the i^{th} component of y is the same as the i^{th}

component of x and/or the i^{th} component of z ; i.e., each component of y is the same as the corresponding component of x and/or z .¹

Examples:

1. Let $x = (..-)$, $y = (.-.)$ and $z = (--.)$.

Since all three signals have 3 components, the first condition of the definition is satisfied. The second condition is also satisfied, since the first component of y is the same as the first component of x , the second component of y is the same as the second component of z , and the third component of y is the same as the third component of x . Therefore, $(.-.)$ is component-wise between $(..-)$ and $(--.)$.

2. Let $x = (.-.-.)$, $y = (....)$, and $z = (-.-.)$.

Since both conditions are satisfied, $(....)$ is component-wise between $(.-.-.)$ and $(-.-.)$.

3. Let $x = (.....)$, $y = (...--)$, and $z = (..---)$.

Since the two conditions are satisfied, $(...--)$ is component-wise between $(.....)$ and $(..---)$.

4. Let $x = (.....)$, $y = (-.....)$, and $z = (...--)$.

Since the first component of y differs from the first component of both x and z , $(-.....)$ is not component-wise between $(.....)$ and $(...--)$.

5. Let $x = (---..)$, $y = (..---)$, and $z = (.----)$.

Since the second component of y differs from the second component of both x and z , $(..---)$ is not component-wise between $(---..)$ and $(.----)$.

¹This is somewhat similar to Restle's (1961) definition of betweenness.

In the sixth definition $d(x)$ denotes the number of dashes in x .

Thus $d(-.--) = 3$.

Definition 6.

For all $x, y, z \in \mu_1$, y is dash-wise between x and z iff either $d(x) \leq d(y) \leq d(z)$ or $d(z) \leq d(y) \leq d(x)$.

Examples:

1. Let $x = (..-)$, $y = (.-)$, and $z = (--.)$.

Since $d(x) < d(y)$, and $d(y) = d(z)$, y is dash-wise between x and z .

2. Let $x = (..-)$, $y = (--.)$, and $z = (..---)$.

Since $d(x) < d(y) < d(z)$, y is dash-wise between x and z . However, since x , y , and z do not all have the same number of components, y is not component-wise between x and z .

3. Let $x = (.....)$, $y = (-.....)$, and $z = (...--)$.

Since $d(x) < d(y) < d(z)$, y is dash-wise between x and z . As we have already stated, however, y is not component-wise between x and z .

4. Let $x = (.-.-.)$, $y = (.....)$, and $z = (-.-.)$.

Although y is component-wise between x and z , y is not dash-wise between x and z . Observe that y has fewer dashes than both x and z .

One can observe that:

(1) y is component-wise between x and z iff y is component-wise between z and x ;

(2) y is dash-wise between x and z iff y is dash-wise between z and x .

(3) y is component-wise and dash-wise between x and z ; and,

(4) if x , y , and z have the same number of components, and x is

closer to y than to z in the ordering on μ_1 , then y is component-wise and dash-wise between x and z .

In the second hypothesis a correspondence between betweenness in these two senses and relative confusability of the signals is proposed.

Hypothesis 2.

For all $x, y, z \in \mu_1$, ($x \neq y \neq z$), if

- (i) y is component-wise between x and z ,
- (ii) y is dash-wise between x and z , and in addition,
- (iii) neither $x < y < z$ nor $z < y < x$,

then x will be confused more often with y than with z .

The third condition was added in order to prevent hypothesis 2 from overlapping with hypothesis 1. Some predictions which follow from the second hypothesis are:

1. (\dots) will be confused more often with (\dots) than with (\dots) , and (\dots) will be confused more often with (\dots) than with (\dots) .
2. (\dots) will be confused more often with (\dots) than with (\dots) , and (\dots) will be confused more often with (\dots) than with (\dots) .
3. (\dots) will be confused more often with (\dots) than with (\dots) , and (\dots) will be confused more often with (\dots) than with (\dots) .

If condition (ii) were omitted, we would predict on the basis of hypothesis 2, that (\dots) would be confused more often with (\dots) than with (\dots) . Intuitively, however, since (\dots) and (\dots) have the same combination of dots and dashes, they appear to be quite similar to each other, perhaps even more similar to each other than (\dots) is to either of these two signals. Thus the second condition prevents us from making such counter-intuitive predictions.

Although many other structural interrelations among the signals could provide the bases for other hypotheses, only these will be considered here. Data obtained by Rothkopf in the experiment described below will be used to test those hypotheses. Since Rothkopf studied only the 36 signals of the International Morse Code, his data can be applied only to that subset of μ_1 .

III. A Test of the First Model

A. Rothkopf's Experimental Design

Procedure. The Ss of this experiment were exposed to pairs of aural Morse signals sent at a high tone speed. The signals of each pair were separated by a short temporal interval. The Ss were asked to indicate whether they thought the two signals were the same (or different) by making the appropriate mark on an IBM True-False Answer sheet. Each S was asked to respond in this fashion to 351 different pairs of Morse signals.

Materials and apparatus. Eight different lists of 351 stimulus pairs each were used in this experiment. The lists were composed in the following manner. The 36 signals of the Morse code allow 1,260 two-signal permutations to which the objectively correct response is "different" and 36 pairs of signals to which the objectively correct response is "same." The 1,260 "different" pairs were divided randomly into four groups of 315 pairs of signals each. All of the 36 "same" pairs were then added to each of the groups of 315 pairs. Each list was in this manner brought to a total of 351 pairs of signals. The entire process described above was repeated to yield four additional lists, for a total of eight lists. Pairs were then drawn at random without replacement for each list of 351 in order to determine order of presentation.

Each of the eight lists was recorded on magnetic tape. The signals were generated by a Boehme Automatic Keyer using a tone speed of 20 words per minute. This tone speed means that each signal was sent fast enough to make 20 transmissions of the word "Codex" within 1 min., or that the temporal duration of each dot was .05 sec., that each dash was .15 sec., and that a silent period between elements was .05 sec. The time interval between the signals of each pair was 1.4 sec., with 3.00 sec. between pairs. To help S keep place on the response sheet an interval of 4.6 sec. was interposed after every tenth pair of signals. A tape-recorded modification of the instructions used with the Signal Corps Code Aptitude Test preceded each list.

The output of a tape recorder carrying the experimental stimuli was, after further amplification, fed to loudspeakers which were mounted in the ceiling of the experimental chamber. This room was judged by E to have good acoustical characteristics. The signals were thought to be of comparable audibility throughout the room. Although the experiment was carried on during the month of July in southern Texas, the experimental room was not air-conditioned.

Subject and design. The Ss of this experiment consisted of 598 airmen awaiting basic training at Lackland AFB. The number was composed of eight separate marching units. These units ranged in size from 51 to 120 men. The Ss who reported Morse code experience were not used. Each of the eight marching units was assigned to a different list of stimulus pairs (Rothkopf, 1957, pp. 95-96).

B. A Restatement of the Hypotheses

In Rothkopf's experiment, as in most auditory experiments, the signals of a pair were presented sequentially rather than simultaneously. Therefore, (x,y), the ordered pair in which x precedes y, is not identical

to (y,x) , the ordered pair in which y precedes x . Since (x,y) and (y,x) are distinct, $p(x,y)$ -- the percentage of "same" responses to the ordered pair, (x,y) -- may be quite different from $p(y,x)$.

There are 36 pairs of Morse Code signals of the form (x,x) . In each of these pairs the first and second signal are the same. Since each signal is structurally more similar to itself than to any other signal, we should expect $p(x,x)$ to be greater than both $p(x,y)$ and $p(y,x)$ for all $x \neq y$.

Rothkopf's data can be used to test the hypotheses of this model, for if it follows from either hypothesis that x will be confused more often with y than with z , then $p(x,y)$ and $p(y,x)$ should both be greater than $p(x,z)$ and $p(z,x)$. In other words, if it is predicted that x will be confused more often with y than with z , then, x and y should be judged to be "same" more often than x and z , irrespective of the order in which the signals are presented.

The hypotheses will now be restated in terms of the percentages of "same" responses to the pairs of Morse Code signals.

Hypothesis 1.

For all $x, y, z \in \text{IMC}$, if x is closer to y than to z in the ordering on μ_1 , then:

- $p(x,y) > p(x,z)$,
- $p(x,y) > p(z,x)$,
- $p(y,x) > p(z,x)$, and
- $p(y,x) > p(x,z)$.

Hypothesis 2.

For all $x, y, z \in \text{IMC}$ ($x \neq y \neq z$), if

- (i) y is component-wise between x and z ,
- (ii) y is dash-wise between x and z , and in addition,
- (iii) neither $x < y < z$ nor $z < y < x$, then

$p(x,y) > p(x,z),$
 $p(x,y) > p(z,x),$
 $p(y,x) > p(z,x),$ and
 $p(y,x) > p(x,z).$

Although the predictions that follow directly from the hypotheses always involve pairs having one signal in common, there are many derivative predictions of the form $p(x,y) > p(w,z)$. For example, since (...) is closer to (-...) than to (--.) in the ordering on μ_1 , it is predicted that $p(..., -...)$ will be greater than $p(..., --.)$; and since (--.) is closer to (...) than to (..) in the ordering on μ_1 , it is predicted that $p(..., --.)$ will be greater than $p(..., ..)$. From these two predictions it follows that $p(..., -...)$ should be greater than $p(..., ..)$. In general, if $w < x < y < z$, then $p(x,y)$ and $p(y,x)$ should both be greater than $p(w,z)$ and $p(z,w)$.

Since there are over 10,000 predictions which follow from the hypotheses, it is desirable to have these predictions organized in some meaningful fashion. In the next section I shall attempt to do this.

C. Simplexes and Double Simplexes in Rothkopf's Data Matrix

Rothkopf's data can be summarized in a 36×36 matrix whose rows indicate the signal presented first, whose columns indicate the signal presented second, and whose cells indicate the percentage of "same" responses to the corresponding ordered pairs. (Rothkopf's data matrix will be shown in the next section of this chapter.) The 36 cells along the main diagonal of such a matrix indicate the percentage of "same" responses to pairs in which the two signals are identical. If it is true that $p(x,x)$ is greater than $p(x,y)$ for all $x \neq y$, then each diagonal cell should be greater than every other cell in its row; and if it is true that $p(x,x)$ is greater than $p(y,x)$ for all $x \neq y$,

then each diagonal cell should be greater than every other cell in its column. The 1260 cells which are not on the main diagonal indicate the percentages of "same" responses to the pairs in which the first and second signals are different, i.e., the percentages of confusions between the signals of these ordered pairs.

Some implications of the hypotheses for the structure of some selected submatrices of Rothkopf's data matrix are as follows.

(1) If $w < x < y < z$, then the percentages in the submatrix whose rows and columns are ordered w, x, y, z, \dots should taper off in the direction above and/or to the right of the main diagonal as well as in the direction below and/or to the left of the main diagonal. A matrix (or submatrix) with such a structure will be referred to as a simplex (Guttman; 1954; 1966).

For example, since $(\dots) < (-\dots) < (--\dots) < (---\dots)$, it is predicted that the submatrix whose rows and columns are ordered $(\dots), (-\dots), (--\dots), (---\dots)$ (or in terms of the alphabetic equivalents, the submatrix whose rows and columns are ordered S,D,Z,Q) will be a simplex. In order for this matrix to have such a structure it is necessary and sufficient that the following inequalities be satisfied:

$$p(S,S) > p(S,D) > p(S,Z) > p(S,Q);$$

$$p(S,S) > p(D,S) > p(Z,S) > p(Q,S);$$

$$p(D,D) > p(D,Z) > p(D,Q);$$

$$p(D,D) > p(Z,D) > p(Q,D);$$

$$p(D,D) > p(D,S);$$

$$p(D,D) > p(S,D);$$

$$p(Z,Z) > p(Z,Q);$$

$$p(Z,Z) > p(Q,Z);$$

$$p(Z,Z) > p(Z,D) > p(Z,S)$$

$$p(Z,Z) > p(D,Z) > p(S,Z)$$

$$p(Q,Q) > p(Q,Z) > p(Q,D) > p(Q,S)$$

$$p(Q,Q) > p(Z,Q) > p(D,Q) > p(S,Q)$$

(2) If (i) $x, y, z, x', y',$ and z' all have the same number of components,

(ii) $x < y < z$ and $x' < y' < z'$,

(iii) $x, y,$ and z have the same combination of dots and dashes as $x', y',$ and z' , respectively, and

(iv) $x, y,$ and z differ from $x', y',$ and z' , respectively on exactly the same components,

then the submatrix whose rows and columns are ordered x, y, z, x', y', z' ; namely,

	x	y	z	x'	y'	z'
x	Submatrix 1			Submatrix 2		
y						
z						
x'	Submatrix 3			Submatrix 4		
y'						
z'						

should be a double simplex (Wish, 1965); i.e.,

(i) the submatrix whose rows and columns are ordered x, y, z (submatrix 1) should be a simplex;

(ii) the submatrix whose rows are ordered x, y, z and whose columns are ordered x', y', z' (submatrix 2) should be a simplex;

(iii) the submatrix whose rows are ordered x', y', z' and whose columns are ordered x, y, z (submatrix 3) should be a simplex;

(iv) the submatrix whose rows and columns are ordered x' , y' , z' (submatrix 4) should be a simplex;

(v) if submatrix 1 is superimposed upon either submatrix 2 or 3, then each cell value in submatrix 1 should be greater than the cell value upon which it is superimposed; and

(vi) if submatrix 4 is superimposed upon either submatrix 2 or 3, then each cell value in submatrix 4 should be greater than the cell value upon which it is superimposed.

Example:

Let $x = (-...)$, $y = (--..)$, $z = (--.-)$

$x' = (..-.)$, $y' = (.-.-)$, and $z' = (.---)$.

Since (i) all six of these signals have four components;

(ii) $x < y < z$ and $x' < y' < z'$;

(iii) x , y , and z have the same combination of components as x' , y' , and z' , respectively; and

(iv) x differs from x' , y differs from y' , and z differs from z' on the same components--the first and third--

it is predicted that the submatrix whose rows and columns are ordered $(-...)$, $(--..)$, $(--.-)$, $(..-.)$, $(.-.-)$, $(.---)$ -- B,Z, Q,F,P,J -- should be a double simplex.

IV. Results and Discussion

A. The Hypotheses

Rothkopf's data appear in Table 2 as well as in Table 3. These two matrices differ only in terms of a permutation of the rows and columns. The cell values in these matrices indicate the percentages of "same" responses given to each ordered pair of Morse Code signals. Thus, 62 per cent of the Ss who were presented the ordered pair (D,B) responded "same."

Table 2 Per Cent "Same" Judgments Obtained by Rothkopf for All Ordered Pairs of Signals from the International Morse Code.

I. Signals Grouped According to Temporal Length

	E	I	T	S	A	N	M	U	R	D	M	S	V	F	L	B	W	K	G	4	6	P	X	C	Z	O	3	7	J	Y	Q	2	8	1	9	0	
• E	97	17	67	07	06	04	04	03	10	06	04	05	03	02	06	13	02	05	04	03	02	01	05	14	05	05	04	01	06	05	03	02	04	03	03	E	
•• I	10	93	16	35	64	30	12	10	19	13	13	06	05	08	16	07	08	05	06	09	08	03	02	07	07	07	09	05	03	05	05	02	02	04	05	I	
••• T	46	14	96	07	13	05	06	08	04	05	06	03	05	03	07	10	04	14	06	03	08	11	02	01	06	06	03	07	06	02	04	05	06	05	14	T	
•••• S	11	16	09	96	17	17	59	56	18	30	05	28	24	26	10	24	12	13	05	15	09	06	10	05	07	06	02	05	03	06	03	02	05	08	05	S	
••••• A	03	46	06	23	92	34	13	37	35	13	25	08	13	14	03	04	17	22	10	05	06	06	12	06	03	06	08	05	05	07	09	07	06	02	02	A	
•••••• N	08	13	10	12	31	93	16	16	28	30	59	04	04	12	08	04	12	16	10	04	06	09	04	13	11	05	03	02	03	06	05	02	02	05	10	N	
••••••• H	09	10	04	37	03	08	87	36	17	25	05	70	59	32	29	45	09	09	08	43	35	14	33	23	11	08	15	17	10	14	08	09	04	03	03	H	
••••~ U	04	21	06	51	14	13	11	93	40	32	11	10	57	32	32	29	34	44	11	34	09	20	17	12	11	06	16	09	07	09	12	06	07	06	04	U	
••••• R	05	07	02	16	13	12	15	23	87	23	14	07	23	34	37	14	62	21	26	04	12	29	14	16	13	12	13	07	12	12	08	10	09	07	01	R	
••••~ D	07	09	07	29	08	07	36	17	45	88	08	09	20	23	56	62	27	81	40	11	19	27	40	17	33	09	06	08	13	15	09	09	10	03	05	D	
••••~ M	07	08	09	07	24	62	08	13	20	14	96	05	04	17	08	12	21	23	29	06	07	10	09	05	08	11	06	11	11	18	15	07	07	05	10	M	
••••• S	02	07	02	27	08	00	67	14	09	14	02	90	45	45	41	45	07	04	04	69	42	13	45	15	10	04	30	24	14	10	07	10	10	14	06	05	S
••••~ V	09	05	06	11	05	01	39	32	17	16	04	44	92	29	43	17	17	26	06	79	36	17	57	24	10	09	28	25	11	35	10	14	10	10	01	05	V
••••~ F	02	05	02	14	04	06	29	21	35	19	07	26	27	90	50	51	25	16	10	25	24	42	19	33	13	10	47	21	33	27	12	16	05	08	05	F	
••••~ L	10	09	05	12	02	02	26	16	28	45	06	26	19	24	86	69	20	27	12	15	29	37	31	43	59	09	17	36	30	25	36	13	16	12	07	L	
••••~ B	05	05	02	18	05	10	21	18	16	31	06	32	34	28	40	84	08	34	17	40	74	22	84	37	42	12	14	43	19	30	25	17	17	12	04	B	
••••~ W	09	04	04	12	09	06	15	19	61	22	15	05	20	36	18	21	86	29	25	16	09	20	22	30	22	26	19	11	25	25	25	22	06	10	03	W	
••••~ K	01	05	02	02	05	12	11	23	22	73	10	08	17	17	33	24	33	91	25	08	18	14	63	38	18	31	17	14	27	16	31	09	13	05	06	K	
••••~ C	01	05	03	05	09	13	06	15	21	38	14	04	14	14	16	18	32	33	90	10	10	52	21	27	39	82	05	17	22	23	23	14	23	15	20	C	
••••~ 4	06	07	03	09	06	03	16	17	11	12	03	42	55	25	19	19	08	13	14	89	44	17	37	19	03	02	44	32	21	24	29	26	10	05	03	4	
••••~ 6	04	02	03	14	07	02	14	12	11	17	06	17	30	23	27	80	09	11	04	24	06	16	58	30	39	07	26	69	11	38	30	14	14	15	05	6	
••••~ P	05	03	04	09	05	06	12	12	23	12	05	15	19	36	46	22	19	14	22	11	17	83	19	33	30	21	24	24	78	41	43	44	23	34	25	13	P
••••~ X	03	01	03	12	07	08	06	12	10	19	10	16	17	28	42	64	21	50	11	27	57	32	91	45	26	24	24	29	35	48	61	20	16	12	17	X	
••••~ C	04	11	06	14	04	03	07	06	24	17	14	10	11	29	35	38	14	24	13	14	30	51	32	87	38	09	31	28	19	82	34	15	24	13	18	C	
••••~ Z	02	07	03	04	03	02	10	09	14	18	11	10	11	22	51	46	12	21	17	09	24	59	36	45	87	15	27	66	23	42	72	21	47	16	15	Z	
••••~ O	05	02	04	03	07	08	07	11	10	06	04	06	14	09	10	07	25	26	76	07	18	37	35	20	27	86	07	14	39	27	35	17	11	19	20	O	
••••~ 3	04	02	06	07	03	02	12	04	09	05	05	23	16	32	13	08	06	06	06	31	41	37	22	21	12	05	86	16	23	25	19	64	17	18	08	3	
••••~ 7	05	06	02	06	06	06	04	03	12	14	07	12	13	25	32	33	09	13	06	15	61	36	30	22	50	07	18	85	24	30	39	29	70	22	20	7	
••••~ J	02	04	07	03	07	05	09	11	09	08	07	07	09	24	31	09	09	22	18	15	11	63	22	38	28	21	33	28	85	32	47	66	29	67	26	J	
••••~ Y	04	01	04	07	09	06	09	06	05	15	05	11	13	26	14	23	21	12	22	15	11	30	44	62	23	14	40	22	30	86	52	44	33	26	23	Y	
••••~ Q	04	05	03	02	08	06	05	06	11	11	07	09	14	15	26	20	12	23	10	12	27	51	37	38	63	22	17	40	27	50	91	32	58	34	37	Q	
••••~ 2	04	25	03	05	07	03	03	06	06	05	02	05	10	20	14	14	11	09	13	20	14	17	37	22	13	17	54	20	26	30	28	89	21	62	16	2	
••••~ 8	03	02	04	06	03	06	06	06	02	06	03	09	07	15	14	23	05	10	15	16	30	24	12	40	50	14	16	60	33	35	45	29	89	42	61	8	
••••~ 1	03	02	02	04	02	07	04	03	09	03	09	10	12	05	14	05	14	05	13	08	08	30	17	10	22	14	13	19	29	19	28	63	32	84	57	35	1
••••~ 9	01	02	07	06	03	11	05	06	05	03	16	04	03	06	07	14	09	06	14	12	11	11	31	23	24	10	09	42	30	21	32	29	56	57	91	78	9
••••~ 0	05	05	04	03	09	03	04	05	02	02	02	05	03	07	07	03	02	08	14	11	22	02	21	11	20	05	09	17	30	15	29	26	52	50	81	94	0
E I T S A N M U R D M S V F L B W K G 4 6 P X C Z O 3 7 J Y Q 2 8 1 9 0																																					

Table 3 Per Cent "Same" Judgments Obtained by Rothkopf for All Ordered Pairs of Signals from the Morse Code.

II. Signals Grouped According to Number of Components

	E	T	I	A	N	M	S	U	R	D	W	K	G	O	N	V	F	L	B	P	X	C	Z	J	Y	Q	5	4	6	3	7	2	8	1	9	0
• • •	11	09	16	17	17	05	96	56	18	30	12	13	05	06	59	24	26	10	24	06	10	05	07	03	06	03	28	15	09	02	05	02	05	08	05	02
• • •	04	06	21	14	13	11	51	93	40	32	34	44	11	06	34	57	32	32	29	20	17	12	11	07	09	12	10	34	09	16	09	06	07	06	04	03
• • •	05	02	07	13	12	14	16	23	87	23	62	21	26	12	15	23	34	37	14	29	14	16	13	12	12	08	07	04	12	13	07	10	09	07	01	02
• • •	07	06	09	08	07	08	29	17	45	88	27	81	40	09	36	20	23	56	16	27	40	17	33	13	15	09	09	11	19	06	08	09	10	03	05	06
• • •	09	04	04	09	06	15	12	19	61	22	86	29	25	26	15	20	36	18	21	20	22	30	22	25	25	25	05	16	09	19	11	22	06	10	03	07
• • •	01	02	05	05	12	10	02	23	22	73	33	91	25	31	11	17	17	33	24	14	63	38	18	27	16	31	08	08	18	17	14	09	13	05	05	06
• • •	01	03	05	09	13	14	05	15	21	38	32	33	90	82	06	14	14	16	18	52	21	27	39	22	23	23	04	10	10	05	17	14	23	15	20	11
• • •	05	04	02	07	08	04	03	11	10	06	25	26	76	86	07	14	09	10	07	37	35	20	27	39	27	35	06	07	18	07	14	17	11	19	20	12
• • •	09	04	10	03	08	05	37	36	17	25	09	09	08	08	87	59	32	29	45	14	33	23	11	10	14	08	70	43	35	15	17	09	04	03	03	03
• • •	09	06	05	05	01	04	11	32	17	16	17	26	06	09	39	92	29	43	17	17	57	24	10	11	35	10	44	79	36	28	25	14	10	10	01	05
• • •	02	02	05	04	06	07	14	21	35	19	25	16	10	10	29	27	90	50	51	42	19	33	13	33	27	12	26	25	24	47	21	16	05	08	05	05
• • •	10	05	09	02	02	06	12	16	28	45	20	27	12	09	26	19	24	86	69	37	31	43	59	30	25	36	26	15	29	17	36	13	16	12	01	03
• • •	05	02	05	05	10	06	18	18	16	31	08	34	17	12	21	34	28	40	84	22	84	37	42	19	30	25	32	40	74	14	43	17	17	12	04	04
• • •	05	04	03	05	06	05	09	12	23	12	19	14	22	21	12	19	36	46	22	83	19	33	30	78	41	43	15	11	17	24	24	44	23	34	25	13
• • •	03	03	01	07	08	10	12	12	10	12	21	50	11	24	06	17	28	42	64	32	91	45	26	35	48	61	16	27	51	24	29	20	16	12	17	06
• • •	04	06	11	04	03	14	14	06	24	17	14	24	13	09	07	11	29	35	38	51	32	87	38	19	82	34	10	14	30	31	28	15	24	13	18	12
• • •	02	03	07	03	02	11	04	09	14	18	12	21	17	15	10	11	22	51	46	59	36	45	87	23	42	72	10	09	25	27	66	21	47	16	15	15
• • •	02	07	04	07	03	08	02	09	11	09	09	22	18	21	05	09	24	31	09	63	22	38	28	85	32	47	07	15	11	33	28	66	29	67	26	23
• • •	04	04	01	09	06	05	07	06	05	15	21	12	22	14	09	13	26	14	23	30	44	62	23	30	86	52	11	15	26	40	22	44	33	26	23	16
• • •	04	03	02	08	06	07	02	06	11	11	12	23	10	22	05	14	15	26	20	51	37	38	63	27	50	91	09	12	26	17	40	32	58	34	37	24
• • •	02	02	07	08	00	02	27	14	09	14	07	04	04	04	67	45	45	41	45	13	45	15	10	14	10	07	90	69	42	30	24	10	10	14	06	05
• • •	06	03	07	06	03	03	09	17	11	12	08	13	14	02	16	55	25	19	19	17	37	19	03	21	24	29	42	89	44	44	32	26	10	05	03	03
• • •	04	03	02	07	02	06	14	12	11	17	09	11	04	07	14	30	23	27	80	16	58	30	39	11	38	30	17	24	86	26	69	14	14	15	05	14
• • •	04	06	02	03	02	05	07	04	09	05	06	06	06	05	12	16	32	13	08	37	22	21	12	23	25	19	23	31	41	86	16	64	17	18	08	10
• • •	05	02	06	06	06	07	06	03	12	14	09	13	06	07	04	13	25	32	33	36	30	22	50	24	30	39	12	15	61	18	85	29	70	22	20	13
• • •	04	03	25	07	03	02	05	06	06	05	11	09	13	17	03	10	20	14	14	37	17	22	13	26	30	28	05	20	14	54	20	89	21	62	16	11
• • •	03	04	02	03	06	03	06	06	02	06	05	10	15	14	06	07	15	14	23	12	24	40	50	33	35	45	09	16	30	16	60	29	89	42	61	26
• • •	03	02	02	02	07	09	04	03	09	03	14	05	13	14	04	12	05	14	05	30	17	10	22	29	59	28	10	08	08	13	19	63	32	84	57	55
• • •	01	07	02	03	11	16	06	06	05	03	08	06	14	10	05	03	06	07	14	31	11	23	24	30	21	32	04	12	11	09	42	39	56	57	91	78
• • •	05	04	05	09	03	02	03	05	02	02	02	08	14	25	04	03	07	03	03	21	12	11	20	30	15	29	05	11	22	09	17	26	52	50	81	94
• • •	E	T	I	A	N	M	S	U	R	D	W	K	G	O	H	V	F	L	B	P	X	C	Z	J	Y	Q	5	4	6	3	7	2	8	1	9	0

One can observe in either Table 2 or Table 3 that, with only one exception, each diagonal cell is greater than every other cell in its row and column. The exception occurs in B's row, for $p(B,X) = p(B,B)$.

Since the values in Tables 2 and 3 tend to decrease in the directions away from the main diagonal, we can anticipate that hypothesis 1 will receive some support. All of the predictions that follow directly from the first hypothesis are based upon a particular relationship among the signals--x is closer to y than to z in the ordering on μ_1 . In some instances y is immediately bounded by x and z, while in other instances this is not the case, i.e., y is not immediately bounded by x and z. On the basis of this dichotomy I shall divide the predictions following from hypothesis 1 into two classes. In the first class are those predictions in which y is immediately bounded by x and z. The predictions in which y is not immediately bounded by x and z are in the second class.

Of the 1240 predictions in the first class 1174, (95%) are correct. Since there are several thousand predictions in the second class, only a sample of 2000 were tested. Of the 2000 predictions 1646, (82%) are in accord with the data. However, of the 354 errors of predictions very few involve a difference greater than 3%. In fact, in 114 of these 354 cases the percentage that was predicted to be greater than another percentage is equal to that other percentage.

In order to get a better perspective of the support for, as well as the limitations on hypothesis 1, it is helpful to study some of the matrices that were predicted to be simplexes. In Table 4 two such matrices are displayed. Observe that the percentages taper off in the directions away from the main diagonal. A close look reveals that within the region of the upper matrix determined by the pair of dotted

TABLE 4

Two Approximate Simplexes in Rothkopf's Data Matrix

		E	I	S	H	5	6	7	8	9	Ø
•	E	97	17	07	04	05	02	04	02	03	03
• •	I	10	93	35	12	06	08	05	02	04	05
• • •	S	11	16	96	59	28	09	05	05	05	02
• • • •	H	09	10	37	87	70	35	18	04	03	03
• • • • •	5	02	07	27	67	90	42	28	10	06	05
• • • • •	6	04	02	14	14	17	86	69	14	05	14
• • • • •	7	05	06	06	04	12	61	85	70	20	13
• • • • •	8	03	02	06	06	09	30	60	89	61	26
• • • • •	9	01	02	06	05	04	11	42	56	91	78
• • • • •	Ø	05	05	03	04	05	22	17	52	81	94

		E	T	A	U	F	C	Y		I	Ø
•	E	97	67	06	03	02	14	06		04	03
• •	T	46	96	13	08	05	01	02		05	06
• • •	A	03	06	92	37	14	06	07		02	02
• • • •	U	04	06	14	93	32	12	09		06	03
• • • • •	F	02	02	04	21	90	33	27		08	05
• • • • •	C	04	06	04	06	29	87	82		13	12
• • • • •	Y	04	04	09	06	26	62	86		26	16
• • • • •	I	03	02	02	03	05	10	19		84	55
• • • • •	Ø	05	04	09	05	07	11	15		50	94

lines there are only two violations of the simplex patterns-- $p(I,E) < p(S,E)$, and $p(6,S) = p(6,H)$; and within the region of the lower matrix determined by the pair of dotted lines there are only four violations of the simplex pattern-- $p(A,E) < p(U,E)$, $p(U,T) = p(A,T)$, $p(C,A) = p(F,A)$, and $p(Y,U) = p(C,U)$. Outside the bounds of the dotted lines, however, there are several violations. If a percentage, $p(x,y)$, is within the dotted lines, then x and y differ in temporal length by no more than 6 units. If a percentage, $p(x,y)$, is outside the bounds of the dotted lines, then x and y differ in temporal length by 8 or more units. It seems that Ss can almost always detect that signals differing by 8 units are different. Any greater difference in temporal length does not appear to make it any easier for Ss to detect the differences among the signals. Thus, there is little differentiation among the percentages (and therefore, several errors of prediction) outside of the dotted lines.

We shall now turn to a test of the second hypothesis. Of the 880 predictions following directly from this hypothesis 772, (88%) are correct. As in the first hypothesis most of the errors of predictions involve small differences in percentages. Since hypotheses 1 and 2 receive considerable support, the matrices shown in Table 5 are approximately double simplexes. Observe in Table 5 that all four 3x3 submatrices of each matrix have simplex patterns. In a double simplex the values in the two submatrices that contain the main diagonal--submatrices 1 and 4--should be greater than the values in the corresponding cells of the other two submatrices--submatrices 2 and 3. Note that in only two instances is a value in submatrix 1 or 4 as small or smaller than the value in the corresponding cell of submatrix 2 or 3-- $p(C,F) < p(C,L)$ and $p(F,C) < p(L,C)$.

TABLE 5

Two Approximate Double Simplexes in Rothkopf's Data Matrix

	B	Z	Q	F	P	J
.... B	84	42	25	28	22	19
... Z	46	87	72	22	59	23
.. Q	20	63	91	15	51	27
... F	51	13	12	90	42	33
.. P	22	30	43	36	83	78
.. J	09	28	47	24	63	85

	L	Z	Q	F	C	Y
... L	86	59	36	24	43	25
... Z	51	87	72	22	45	42
.. Q	26	63	91	15	38	50
... F	50	13	12	90	33	27
.. C	35	38	34	29	87	82
.. Y	14	23	53	26	62	86

B. Other Patterns of Confusion Among the Morse Code Signals

Having tested the hypotheses, I shall now report some additional observations about the patterns of confusion among Morse Code signals. I shall begin by examining the most common types of confusions, ("same" responses to pairs in which the signals are different).

The most habitual kind of confusion is the failure to detect a difference between signals that have the same number of components but differ on a single component. For example, $p(....., ...-) = 59\%$ and $p(...-,) = 39\%$. There are 88 ordered pairs in which the two signals have the same number of components, but differ on one component. In all pairs in this category the signal with more dashes is a successor of the other signal. The mean confusion rate for these 88 ordered pairs is 47.3%. There is, however, considerable variability in the confusion rate as a function of the order in which the signals of the pair are presented and the particular component on which the signals differ. If y is a successor of x , and x and y have the same number of components, then in 38 of the 44 pairs, $p(x,y)$ is greater than $p(y,x)$; e.g., $p(....., ...-)$ is greater than $p(...-,)$. The mean of these $p(x,y)$'s is 52%, whereas the mean $p(y,x)$ is 42.6%. Thus, on the average, there are 9.4% more confusions when the shorter signal is presented first than when the longer signal is presented first.

Table 6 presents an analysis of the percentage of "same" judgments as a function of the number of components and the particular component on which the signals differ. We can observe in this table that there is a greater likelihood that Ss will respond "same" if the difference between the signals occurs on the final component than if the difference is on any other component. An explanation for the effect is that the duration of the final component is more ambiguous or indefinite than

TABLE 6

Percentage of "Same" Responses to Pairs of
Signals Which Differ on A Single Component

Component on Which Signals Differ	Number of Components				
	1	2	3	4	5
Final	56.5	57.8	67.8	66.6	67.5
Fourth (but not final)					48.0
Third (but not final)				38.9	62.0
Second (but not final)			27.8	39.9	63.8
First (but not final)		23.0	28.0	39.0	41.0

that of any other component, since the cue concerning the duration of a component provided by the onset of a subsequent component is unavailable. Another thing to note in Table 6 is that as the number of components increases, it becomes harder to detect that signals differ on a particular component.

The second most frequent kind of confusion is with regard to pairs in which the two signals have the same combination of dots and dashes, but differ in terms of an inversion of two components. In this category are confusions among signals of such pairs as (\dots , \dots) and (\dots , \dots). The average percentage of "same" judgments for the 46 ordered pairs in this category is 36.3%. That signals differing in terms of an inversion of two components are called "different" 63.7% of the time indicates that the precise sequence of dots and dashes, as well as the number of dots and dashes, is of some importance in the judgmental process.

The next most common kind of confusion is the failure to observe that one signal of a pair has one more dot than the other. In order for confusions to be in this category, the signal with the additional dot must be a successor of the other signal. The mean percentage of "same" judgments for the 64 pairs in this category is 32.4%.

The effect which order of presentation has on the percentage of "same" judgments for pairs in this category is even stronger than that reported for signals differing on a single component. In 29 of the 32 cases, if x can be transformed to y by adding a dot to x , then $p(x,y) > p(y,x)$. For example, $p(\dots, \dots) = 52\%$ whereas $p(\dots, \dots) = 22\%$. The mean $p(x,y)$ is 39.3%, whereas the mean $p(y,x)$ is 25.4%.

We have seen that if y is a successor of x , then in 67 of the 76 instances $(38 + 29 / 44 + 32)$, $p(x,y)$ is greater than $p(y,x)$. This is part of a more general effect of order of presentation -- if $x < y$, then

$p(x,y)$ tends to be greater than $p(y,x)$. However, as the differences in temporal length between x and y increases, this nonsymmetry decreases.

The effect associated with order of presentation is similar to the time-error phenomenon which Postman (1946) observed in loudness discrimination. When the stimuli of a pair were separated by a 1 to 2 second interval, Postman reports that the loudness of the second signal tended to be underestimated. In Rothkopf's experiment the Ss appear to underestimate the duration of and/or number of components in the second signal. Since presenting the shorter signal first attenuates the difference between the signals and presenting the longer signal first exaggerates the difference between the signals, Ss respond "same" more often when the shorter signal precedes the longer signal.

The presentation-order, or "time-error," phenomenon has particular relevance to hypothesis 1. If $x < y < z$, then the following predictions follow from hypothesis 1:

- | | |
|-----------------------|-----------------------|
| (1) $p(x,y) > p(x,z)$ | (5) $p(z,y) > p(z,x)$ |
| (2) $p(x,y) > p(z,x)$ | (6) $p(z,y) > p(x,z)$ |
| (3) $p(y,x) > p(z,x)$ | (7) $p(y,z) > p(x,z)$ |
| (4) $p(y,x) > p(x,z)$ | (8) $p(y,z) > p(z,x)$ |

According to the presentation-order phenomenon described above if $x < y < z$, then:

- (9) $p(x,y) > p(y,x)$,
- (10) $p(y,z) > p(z,y)$,
- (11) $p(x,z) > p(z,x)$.

These 11 inequalities are combined into the following orderings among the percentages:

$$p(x,y) > p(y,x) > p(x,z) > p(z,x);$$

$$p(y,z) > p(z,y) > p(x,z) > p(z,x).$$

One can observe in Rothkopf's data that if $x < y < z$, then (for a given difference in temporal length between x and y , and between y and z), violations of hypothesis 1 are most likely to occur when $p(y,x)$ or $p(z,y)$ is compared with $p(x,z)$, and violations are least likely to occur when $p(x,y)$ or $p(y,z)$ is compared with $p(z,x)$. In the former cases the "time-error" phenomenon increases the probability of an error of prediction, whereas in the latter cases, the "time-error" phenomenon decreases the probability of an error of prediction. Note above that the difference between $p(x,y)$ and $p(z,x)$ is usually greater than the difference between $p(y,x)$ and $p(x,z)$, and that the difference between $p(y,z)$ and $p(z,x)$ is usually greater than the difference between $p(z,y)$ and $p(x,z)$.

Shepard (1963), in comparing Rothkopf's data with substitution error data obtained by Keller and Taubman (1943) and by Plotkin (1943), points out that the patterns of confusion among Morse Code signals differ considerably from the patterns of substitution errors made by Ss learning the International Morse Code. For example, signals that can be transformed into each other by converting all dots to dashes and vice versa, such as (...) and (---), or (-...) and (.---), are frequently substituted for each other during learning despite the fact that they are rarely confused when presented in immediate succession. Shepard (1963, p. 43) concludes that "the learning task -- unlike Rothkopf's short-term comparison task -- requires the establishment of long-term associative connections. Perhaps in this latter situation, then, the Ss are still able to perceive whether three dots or three dashes were presented. Apparently they can also remember that the two responses assigned to these signals are "S" and "O." What they cannot keep straight is which response goes with which signal." On the basis of the patterns of substitution errors in Plotkin's

and in Keller and Taubman's data, it can be concluded that in this model the number of times each signal is substituted for every other signal cannot be used as a measure of signal confusability.

C. Multidimensional Configurations for the Morse Code Signals

Having discussed some of the most obvious patterns in Rothkopf's data, let us now consider some geometric representations of the data matrix which will make the total structure more comprehensible. All of the configurations which will be presented were obtained by means of nonmetric multidimensional scaling procedures (Kruskal, 1964a, 1964b; Guttman, 1967, in preparation; Lingoes, 1966b, in press). The aim of these procedures is to obtain a configuration of points in a Euclidean space² of a specified number of dimensions such that distances between points in the space are monotonic with some set of coefficients. Since the coefficients in Rothkopf's matrix, the percentages, can be assumed to be measures of similarity (rather than dissimilarity), the distances should be a monotonic decreasing function of these coefficients. The stress of the configuration (Kruskal, 1964a), or the coefficient of alienation (Lingoes, 1966a), is a normalized residual sum of squared deviations from perfect monotonicity. Thus the lower the stress, or coefficient of alienation, the more adequately the configuration reflects the structure of the data matrix.

Shepard (1963) obtained the configuration shown in Fig. 1 (stress = .18) by applying Kruskal's (1964) program (using the symmetric option) to Rothkopf's data. The input to the computer was the symmetric matrix appearing on page 97 of Rothkopf's (1957) paper. In order to obtain the symmetric matrix Rothkopf averaged the two symmetrically situated cells

²The Kruskal program can give a solution in terms of any specified Minkowski ℓ -metric (Kruskal, 1964b).

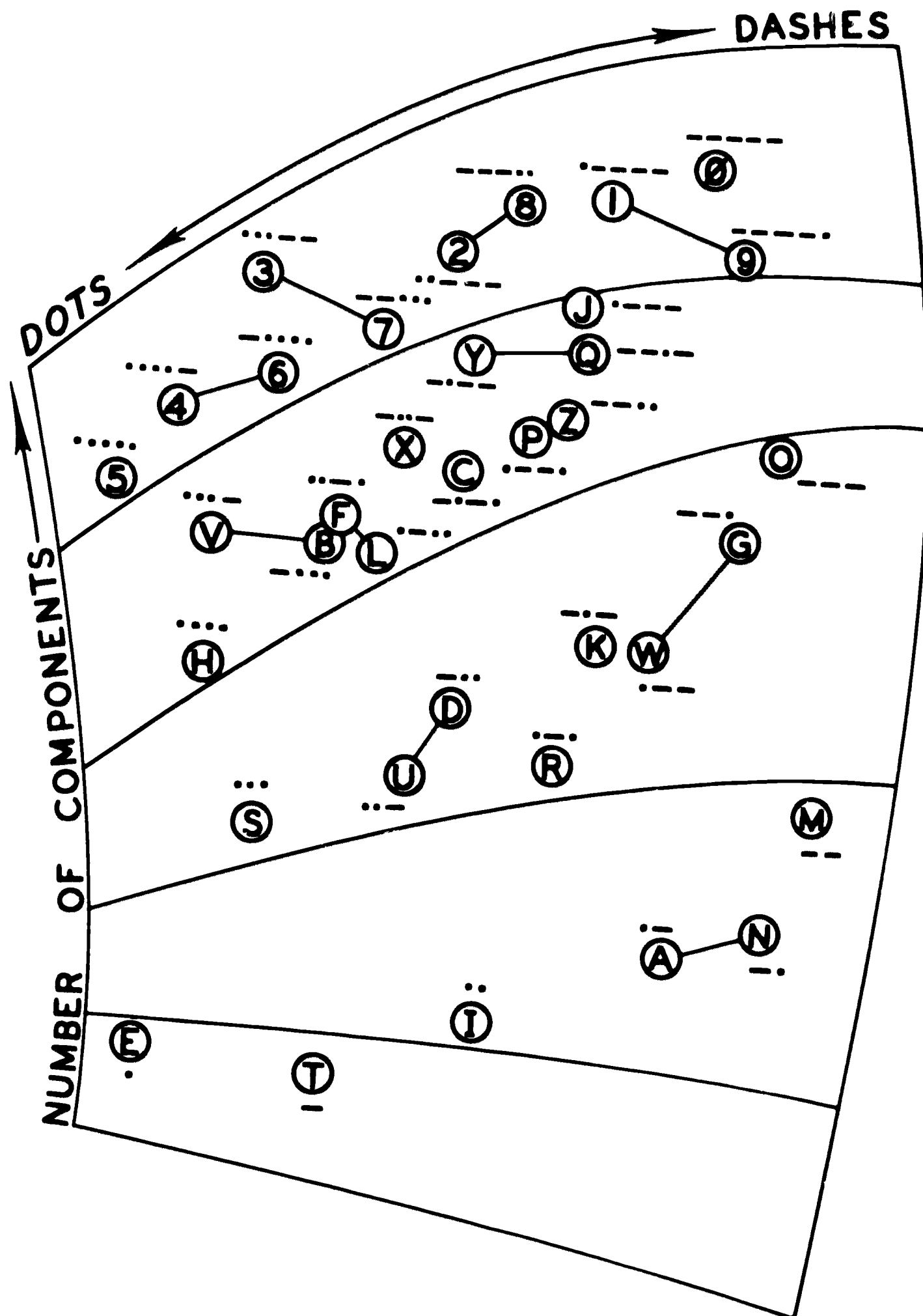


Fig. 1. A two-dimensional configuration of all 36 International Morse Code signals based upon the same-or-different judgments obtained by Rothkopf - I. [Kruskal symmetric solution - from Shepard, 1963, p. 39]

of the original nonsymmetric matrix. Shepard (p. 39) describes his configuration as follows:

The Centers of the circles give the positions of the 36 points as specified by the 72 coordinates of the final solution. The curvilinear lines are not part of this solution per se; these lines were subsequently added to facilitate communication of the interpretation that I am inclined to place on the total configuration of points. The symbol inscribed in each circle is the one assigned to the corresponding dot-and-dash pattern in the Morse Code. The dot-and-dash pattern itself is indicated just outside the circle.

As you can see, I have used the lines to divide the roughly rectangular region of the solution into five more or less parallel strips or bands. All signals within any one band contain the same number of elementary components (i.e., dots and/or dashes). Moreover these five bands occur in order, with respect to their associated number of components from the bottom band, which contains the two one-component signals, to the top band, which contains the ten five-component signals. Clearly then, the axis that cuts across these five bands in this two-dimensional configuration defines a dimension of number of components....

The interpretation of the second dimension seems equally clear. If we examine the disposition of signals within any one band, we immediately see that dots predominate on the left and dashes on the right. Moreover, the ratio between the numbers of these two types of components steadily shifts in the same direction as we traverse the length of any one band.

Let us examine Fig. 1 carefully to see if we can observe any other patterns in this configuration. In the description of the most

common kinds of errors made by Ss, it was pointed out that signals are more likely to be confused if they differ on the last component than if they differ on any other component. We should, therefore, expect signals that differ on the last component to be closer to each other than signals that differ on any other component. Figure 1 shows that this is generally true.

The configuration appearing in Fig. 2 is the same as that shown in Fig. 1. The only difference in the figures is in terms of the boundary lines. With few exceptions if $x < y$, then y is above x in Figs. 1 and 2. More generally, signals having the same temporal length are in the same band; straight line boundaries separate signals having different temporal lengths; and the bands are ordered according to temporal length. Signals that fall between an adjacent pair of diagonal lines generally have the same number of components. There are a few violations of these patterns, however.

The signals within each region determined by the intersections of the horizontal and diagonal lines have the same number of components and temporal length; therefore, they have the same combination of dots and dashes. One can observe that sets of signals whose corresponding submatrices are predicted to be simplexes (as those displayed in Table 4) fall along arcs going from the lower left-hand corner to the upper right-hand corner of the figure.

Now let us look at Figs. 3 and 4. Although these configurations are structurally very similar to the configurations we have discussed, they were obtained by nonsymmetric, rather than symmetric procedures. By suitable rotations (which do not alter the configuration) the similarity of these configurations becomes even more apparent. As described below the figures, the configuration of Fig. 3 is the solution

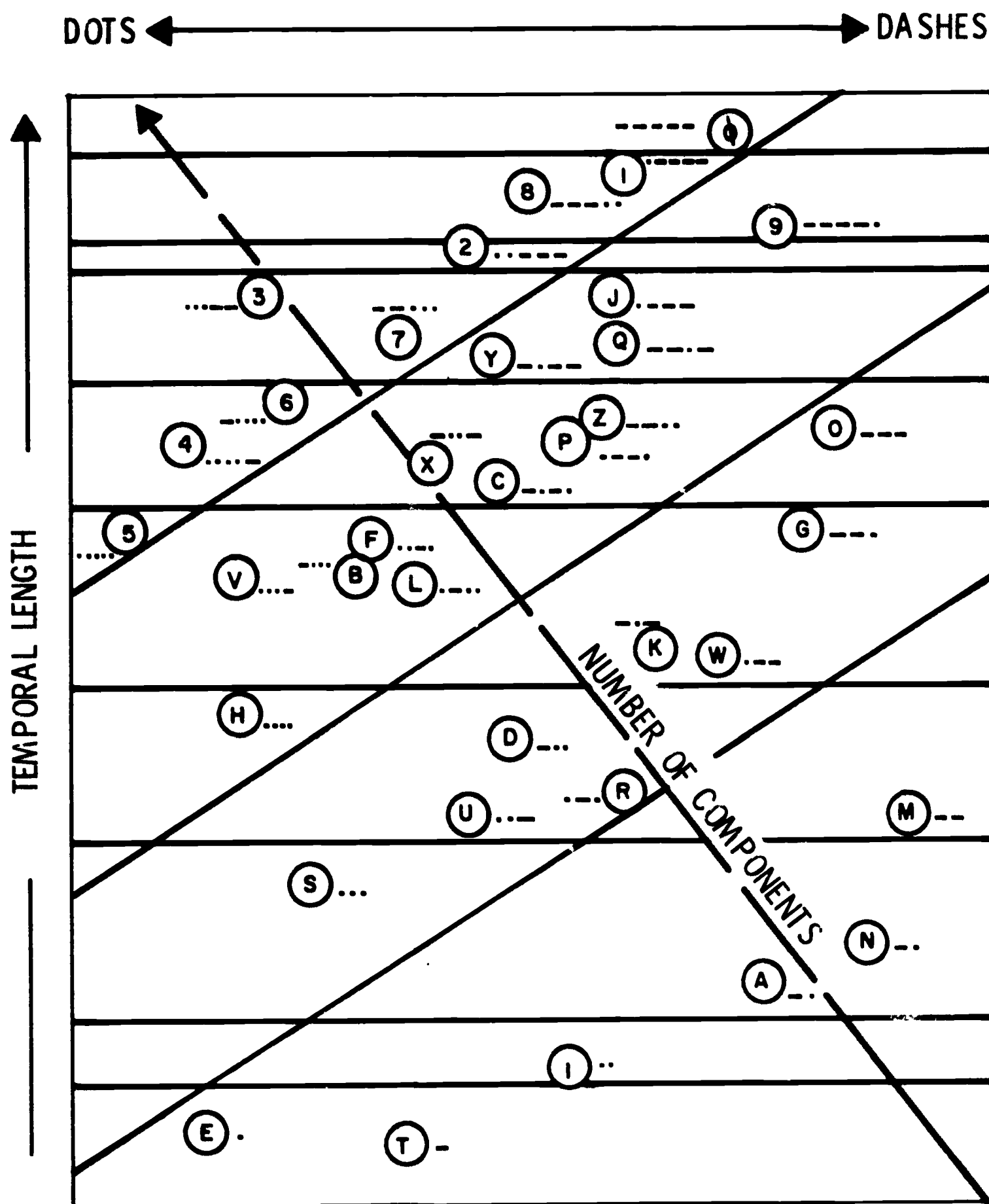


Fig. 2. A two-dimensional configuration of all International Morse Code signals based upon the same-or-different judgments obtained by Rothkopf - II. [Kruskal symmetric solution - from Shepard, 1963, p. 39]

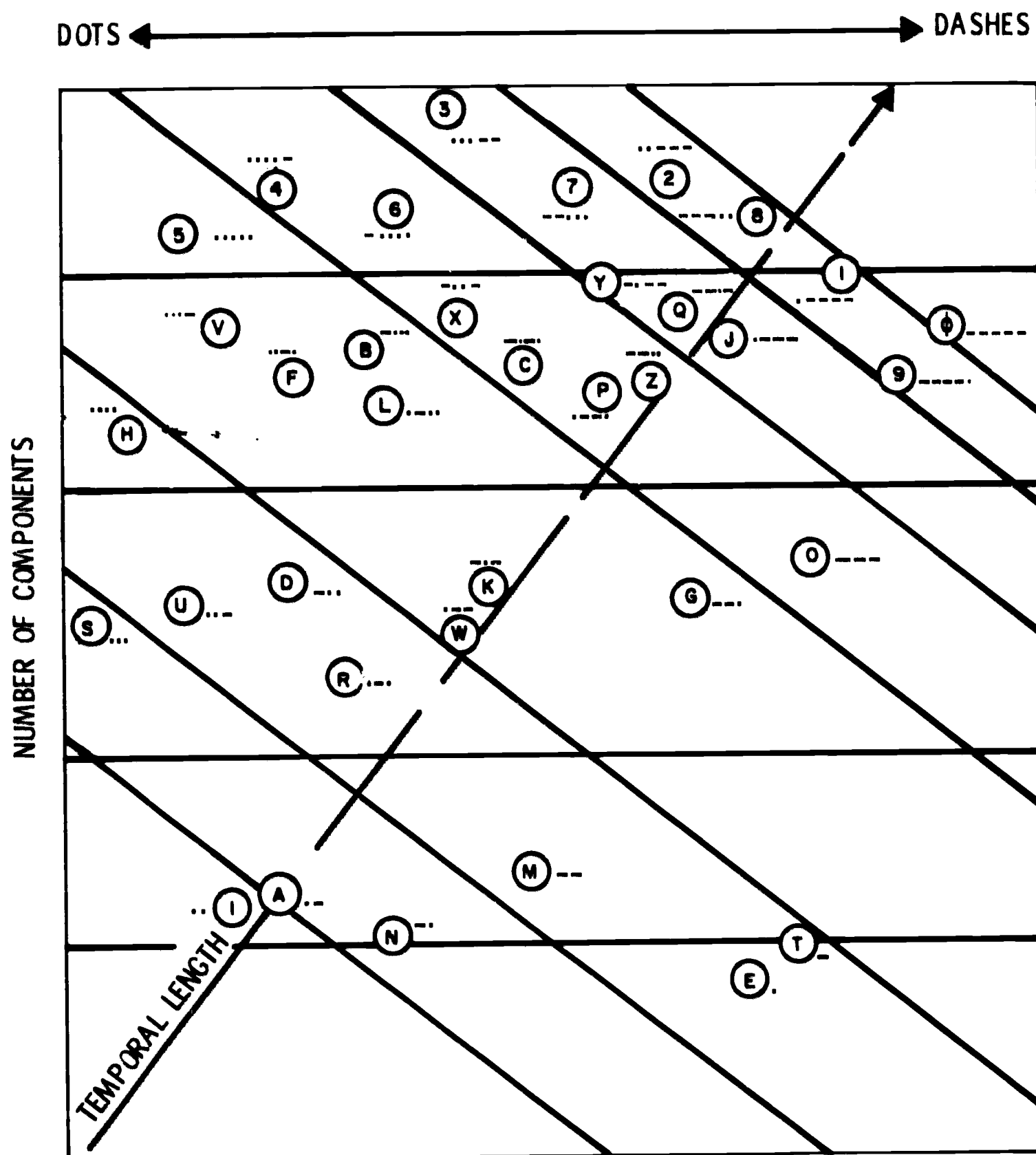


Fig. 3. A two-dimensional configuration of all International Morse Code signals based upon the same-or-different judgments obtained by Rothkopf - III. [Kruskal non-symmetric solution]

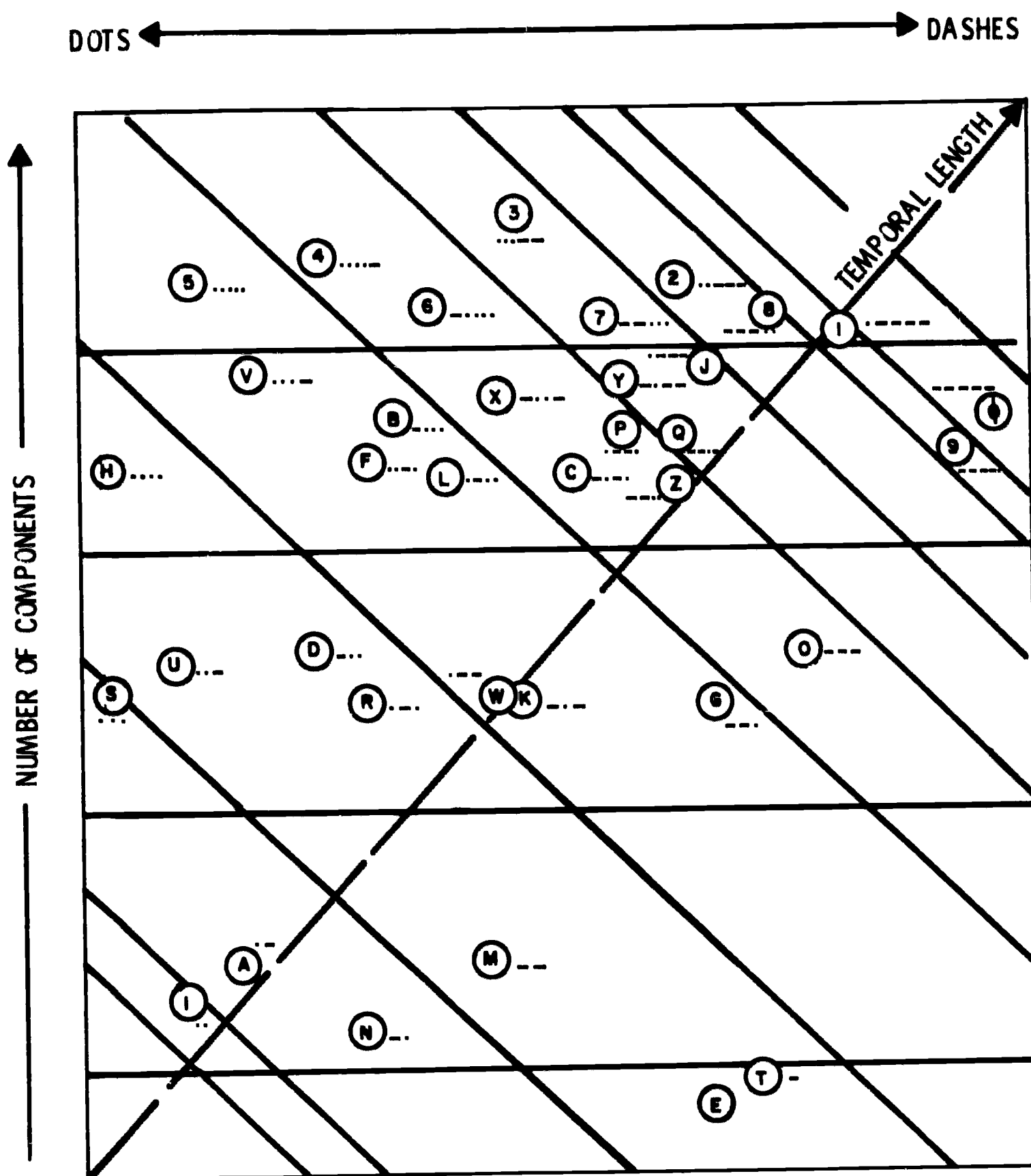


Fig. 4. A two-dimensional configuration of all International Morse Code signals based upon the same-or-different judgments obtained by Rothkopf - IV. [Guttman-Lingoes (Smallest Space Analysis - II) row solution]

(stress = .23) obtained by applying the Kruskal procedure with the nonsymmetric option, whereas the configuration of Fig. 4 is the row solution (coefficient of alienation = .18) obtained by means of G-L(SSA-II) of Guttman and Lingoes (Lingoes, 1965b). In the Kruskal procedure (symmetric or nonsymmetric option) a solution is sought which will best satisfy all inequalities of the form $D_{i,j} \leq D_{k,l}$ (where $D_{i,j}$ is the distance between points i and j in the configuration) if and only if $p(i,j) \geq p(k,l)$. Thus all cells in the matrix are assumed to be comparable. In the Guttman-Lingoes row solution only cells within the same row are assumed to be comparable. Thus one attempts only to satisfy inequalities of the form $D_{i,j} \leq D_{i,k}$ if and only if $p(i,j) \geq p(i,k)$. In the Guttman-Lingoes column solution one attempts to satisfy all inequalities of the form $D_{j,i} \leq D_{k,i}$ if and only if $p(j,i) \geq p(k,i)$. The rationale for using G-L(SSA-II) is that there is a systematic nonsymmetry in the matrix. In order for the Kruskal procedure to be entirely legitimate, the departure from symmetry should be due to statistical fluctuation (Kruskal, 1964a; p. 21). For a further discussion concerning the differences between the Guttman-Lingoes and Kruskal procedures, see Lingoes (1966b, in press).

Figure 5 shows the first two dimensions of the 3-dimensional G-L(SSA-II) row solution (coefficient of alienation = .13). This figure is quite similar to the other figures. The third dimension of this configuration is provided in Table 7. Observe that the signals of a given temporal length that begin with a dash are always higher on the third dimension than those of that temporal length that begin with a dot. Furthermore, if the first dash of one signal occurs on an earlier component than the first dash of another signal which has the same temporal length, then the first signal is higher than the

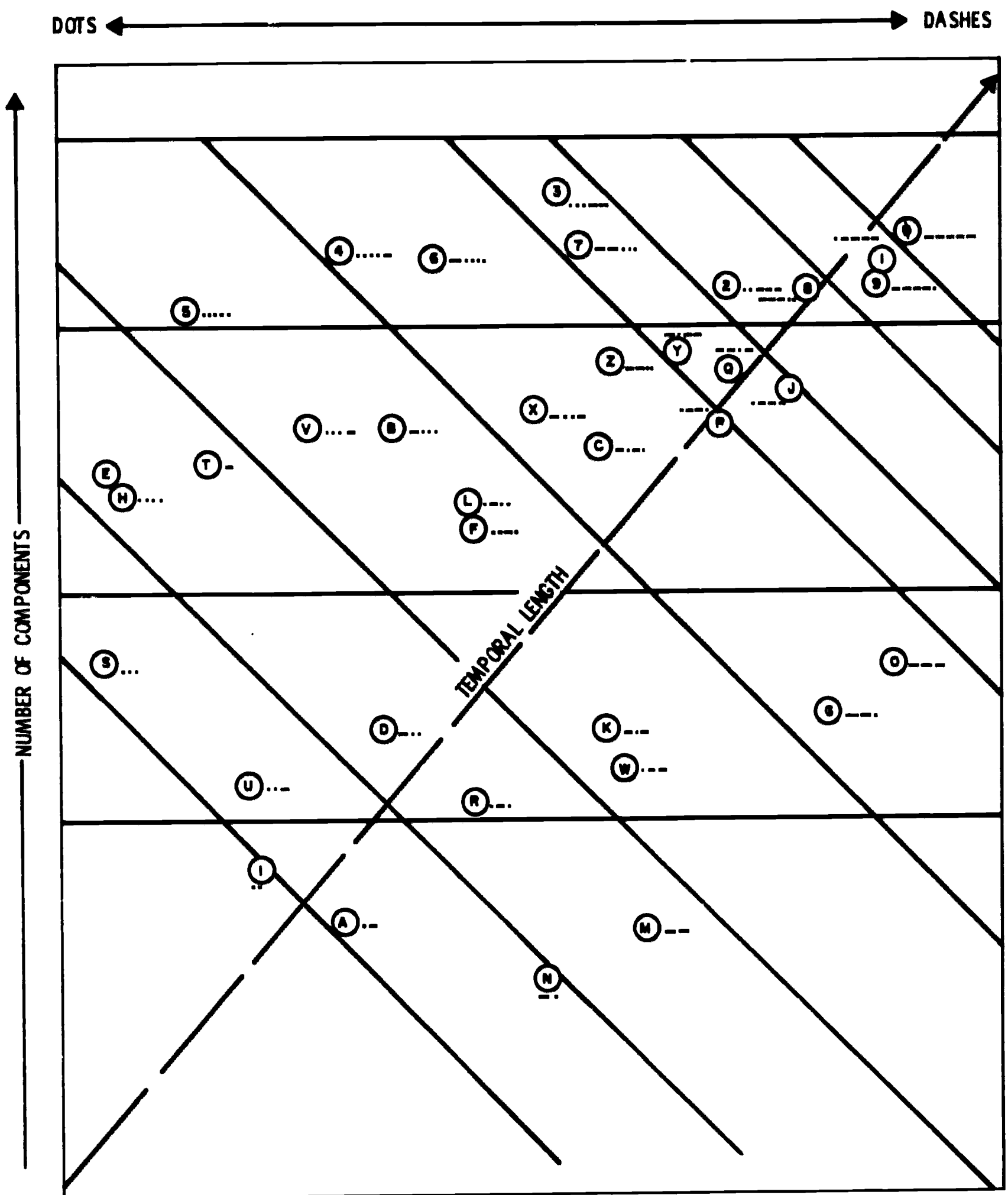


Fig. 5. First two dimensions of a three-dimensional analysis of Rothkopf's data. [Guttman-Lingoes (Smallest Space Analysis - II) row solution]

TABLE 7

Third Dimension of the Three-Dimensional G-L(SSA-II) Row Solution

Signal	Temporal Length in Units	Projection on Third Dimension	Signal	Temporal Length in Units	Projection on Third Dimension
.	1	1350	11	185
-	3	1510	-. . . .	11	120
. .	3	750	- - - -	11	-120
.	5	725	- . . .	11	-120
. . -	5	510	- . . .	11	-360
. . .	5	0 -	11	-505
- - -	7	665 -	11	-580
- . .	7	-325	- - . . .	13	75
. - .	7	-400	- - . - -	13	-115
. . . -	7	-430	- . - - -	13	-280
. . . .	7	-450 -	13	-420
- . . .	9	-110 -	13	-630
- - . .	9	-140	- - - . .	15	185
- . - -	9	-250	. . - - -	15	-375
. - . . .	9	-365	- - - . .	17	570
.	9	-525	. - - - -	17	-240
. . . . -	9	-775	- - - - -	19	540
. . . . -	9	-820			
. . . . -	9	-840			

second signal on the third dimension. Thus the third dimension groups signals of a given temporal length on the basis of the particular sequence in which the components occur.

By means of these multidimensional configurations we can comprehend the basic pattern of interrelations latent in Rothkopf's data matrix. We can observe that the temporal length, the number of components, the relative number of dashes, and the precise sequences of dots and dashes in these signals are all relevant to the perceptual-judgmental process. As a step toward developing a more general theory for pattern perception, a model for another signal universe will be proposed. In the next chapter I shall describe explicitly the signal universe of the second model, propose some hypotheses about the relative confusability of these signals, and test these hypotheses.

CHAPTER 3. THE SECOND MODEL

I. The Universe of Content

The signals studied in this model are identical with the 3-component International Morse Code signals in terms of the combinations of dots and dashes. However, whereas in the IMC signals (as well as in all other signals in μ_1) the silent intervals between each pair of adjacent tones are always one unit in duration, in the signals of this model the silent intervals (like the tones) can be either one unit or three units in duration. The signals in this universe can be described as rhythmic patterns, since varying the silences affects the rhythm of the signals.

Formally, the signal universe for the second model, μ_2 , is equal to $\Delta \times \Sigma \times \Delta \times \Sigma \times \Delta$, where Δ is a set whose elements are a 1-unit tone (a dot) and a 3-unit tone (a dash), and Σ is a set whose elements are a 1-unit silence (a short silence) and a 3-unit silence (a long silence). Since Δ and Σ have two elements each, $\Delta \times \Sigma \times \Delta \times \Sigma \times \Delta$ has $2^5 = 32$ elements. Eight of these elements, (the signals having two short silences, i.e., silent intervals) are also in the International Morse Code. Thus $\mu_1 \cap \mu_2 = \{(\dots), (\dots-), (\dots-), (\dots-), (\dots-), (\dots-), (\dots-), (\dots-)\}$.

Because the silent intervals in the signals of μ_1 , were always one unit long, there was no need to indicate explicitly the duration of each intercomponent silent interval. In these Morse-like rhythmic patterns, however, a 1 or a 3 will be written between a pair of tones in order to indicate whether there is one or three units of silence, respectively, between these tones. For example, in $(.1.3-)$ the first silence is 1-unit in duration, while the second is three units in duration. This signal differs from the IMC signal $(\dots-)$, which will be represented here as $(.1.1-)$, only in terms of the duration of the second silence.

In Table 8 the signals in μ_2 are listed along with a set of alphabetic equivalents, each of which has a subscript. All signals with the same dot-and dash, or tonal, pattern have been assigned the same letter, the letter corresponding to that pattern in the International Morse Code. All signals with the same pattern of silent intervals have been assigned the same subscript. The signals whose alphabetic equivalents have 1's as subscripts are the eight 3-component signals of the International Morse Code.

In discussing the signals of this model it will be convenient to refer to the silences, as well as the dots and dashes, as being components of the signals. Each signal, then, has five components; the first, third, and fifth components are tones, whereas the second and fourth components are silences.

II. The Structural Hypotheses

Although the hypotheses of this model are quite similar to those of the first model, it will be necessary to modify some of the previous definitions and to define some additional terms.

A. The First Hypothesis

Definition 1.

For all, $x, y \in \mu_2$, $x < y$ iff the duration of each component of x is less than or equal to the duration of the corresponding component of y , and the duration of at least one component of x is less than the duration of the corresponding component of y , i.e., iff x is weakly dominated by y .

For example, $(.1.1.) < (.1.3.) < (.1-3.) < (-1-3.) < (-3-3.) < (-3-3-)$. On the other hand $(.1.3-)$ and $(-1-3.)$ are not comparable--neither $(.1.3-) < (-1-3.)$ nor $(-1-3.) < (.1.3-)$ --since each signal exceeds the other on some component. One can observe that the ordering among the eight

TABLE 8

The Signal Universe for Model 2

Signal	Alphabetic Equivalent	Temporal Length in Units	Signal	Alphabetic Equivalent	Temporal Length in Units
• • • • •	S ₁	5	• 1 - 1 -	W ₁	9
• • • 3 •	S ₂	7	• • - 3 -	W ₂	11
• 3 • • •	S ₃	7	• 3 - 1 -	W ₃	11
• 3 • 3 •	S ₄	9	• 3 - • -	W ₄	13
• 1 • 1 -	U ₁	7	- • • • -	K ₁	9
• 1 • 3 -	U ₂	9	• • • • •	K ₂	11
• 3 • 1 -	U ₃	9	- 5 • • •	K ₃	11
• 3 • 3 -	U ₄	11	- 3 • • -	K ₄	13
• 1 • • •	R ₁	7	- 1 - 1 •	G ₁	9
• 1 - 3 •	R ₂	9	- 1 - 3 •	G ₂	11
• • - 1 •	R ₃	9	- 7 - • •	G ₃	11
• 3 - 7 •	R ₄	11	- 3 - 3 •	G ₄	13
- • • • •	D ₁	7	• • • 1 •	O ₁	11
- 1 • • •	D ₂	9	• 1 • • •	O ₂	13
- • • 1 •	D ₃	9	- 3 • • -	O ₃	13
- • • • •	D ₄	11	- 3 - 3 -	O ₄	15

signals of the IMC determined by this definition is identical to the ordering obtained for these signals in the first model.

Definition 2.

For all $x, y \in \mu_2$, y is an immediate successor of x iff $x < y$, and there is no $z \in \mu_2$ such that $x < z < y$.

It follows from this definition that y is a successor of x iff x can be transformed to y by changing one of x 's dots to a dash or by changing one of x 's short silences to a long silence.

Definition 3.

For all $x, y, z \in \mu_2$, y is immediately bounded by x and z iff one of the following is true:

(1) y is an immediate successor of x , and z is an immediate successor of y ; or

(2) y is an immediate successor of z , and x is an immediate successor of y .

Thus since $(-1-3.)$ is an immediate successor of $(.1-3.)$ and $(-3-3.)$ is an immediate successor of $(-1-3.)$, $(-1-3.)$ is immediately bounded by $(.1-3.)$ and $(-3-3.)$, or equivalently, $(-1-3.)$ is immediately bounded by $(-3-3.)$ and $(.1-3.)$.

Definition 4.

For all $x, y, z \in \mu_2$, x is closer to y than to z in the ordering on μ_2 iff $x < y < z$ or $z < y < x$.

Hypothesis 1.

For all $x, y, z \in \mu_2$, if x is closer to y than to z in the ordering on μ_2 , then x will be confused more often with y than with z .

B. The Second Hypothesis

Before stating the second hypothesis, I shall define four kinds of betweenness for signals, all of which are concerned with temporal

properties of the signals. In these definitions $d(x)$, $s(x)$, and $t(x)$ denote the number of dashes in x , the number of long silences in x , and the temporal length of x , respectively.

Definition 5.

For all $x, y, z \in \mu_2$, y is component-wise between x and z iff for all i , the i^{th} component of y is the same as the i^{th} component of x and/or the i^{th} component of z .

Definition 6.

For all $x, y, z \in \mu_2$, y is dash-wise between x and z iff either $d(x) \leq d(y) \leq d(z)$ or $d(z) \leq d(y) \leq d(x)$.

Definition 7.

For all $x, y, z \in \mu_2$, y is silence-wise between x and z iff either $s(x) \leq s(y) \leq s(z)$ or $s(z) \leq s(y) \leq s(x)$.

Definition 8.

For all $x, y, z \in \mu_2$, y is time-wise between x and z iff either $t(x) \leq t(y) \leq t(z)$ or $t(z) \leq t(y) \leq t(x)$.

If y equals x or z , or if x is closer to y than to z in the ordering on μ_2 , then y is component-wise, dash-wise, silence-wise, and time-wise between x and z . However, it is not necessary for y to be equal to x or z , or for x to be closer to y than to z in the ordering on μ_2 for y to be between x and z in all four senses. Some examples are as follows:

1. Let $x = (.3.3.)$, $y = (-3.1.)$, and $z = (-1.1-)$.
2. Let $x = (-1-3.)$, $y = (.1-3-)$, and $z = (.1.3-)$.
3. Let $x = (.1.3-)$, $y = (.3.1-)$, and $z = (.3-1-)$.

In all three examples y is component-wise, dash-wise, silence-wise and time-wise between x and z . However, in each of these examples x , y , and z are distinct and x is not closer to y than to z in the ordering on

μ_2 . It is important to note that it is possible for y to be between x and z in any three of these ways without being between x and z in the remaining way. These examples illustrate this point.

1. Let $x = (-1.1.)$, $y = (-3.1.)$, and $z = (-1.3.)$.

Although y is dash-wise, silence-wise, and time-wise between x and z , y is not component-wise between x and z -- the second component of y differs from the second component of both x and z .

2. Let $x = (.1-3.)$, $y = (.3.3.)$ and $z = (-3.3.)$.

Although y is component-wise, silence-wise, and time-wise between x and z , y is not dash-wise between x and z . Observe that y has fewer dashes than both x and z .

3. Let $x = (.1-3-)$, $y = (-3.3.)$, and $z = (-3.1.)$.

The only sense in which y is not between x and z is silence-wise betweenness--the number of long silences in y is greater than the number of long silences in both x and z . Observe that y is component-wise, dash-wise, and time-wise between x and z .

4. Let $x = (-1-1-)$, $y = (.1-1-)$, and $z = (.3.3-)$.

Although y is component-wise, dash-wise, and silence-wise between x and z , y is not time-wise between x and z -- the temporal length of y is 9 units, whereas the temporal length of both x and z is 11 units.

Hypothesis 2.

For all $x, y, z \in \mu_2$, $(x \neq y \neq z)$, if

- (i) y is component-wise between x and z ,
- (ii) y is dash-wise between x and z ,
- (iii) y is silence-wise between x and z ,
- (iv) y is time-wise between x and z , and in addition
- (v) neither $x < y < z$ nor $z < y < x$,

then x will be confused more often with y than with z .

For example, it is predicted that:

1. (.3.3.) will be confused more often with (-3.1.) than with (-1.1-), and that (-1.1-) will be confused more often with (-3.1.) than with (.3.3.);
2. (-1-3.) will be confused more often with (.1-3-) than with (.1.3-), and that (.1.3-) will be confused more often with (.1-3-) than with (-1-3.); and
3. (.1.3-) will be confused more often with (.3.1-) than with (.3-1-), and that (.3-1-) will be confused more often with (.3.1-) than with (.1.3-).

Having stated the hypotheses, I shall now describe an experiment which was designed expressly for testing the second model.

III. A Test of the Second Model

A. Experimental Design

Three-hundred-twenty-four Ss, all of whom were female undergraduates at the University of Michigan enrolled in an introductory psychology course,³ participated without pay in the experiment. None of the Ss had had any prior training in Morse Code. The Ss, who were run in nine groups of 36 Ss each, were presented ordered pairs of the Morse-like rhythmic patterns and were asked to indicate on an IBM True-False Answer Sheet whether they thought that the two signals of each pair were the same or different. Six groups of Ss responded to 363 signal pairs; the remaining three groups responded to 362 pairs.

³The only reason for using all female Ss was that there were no male Ss available in the S pool.

Six lists of 363 signal pairs each and three lists of 362 signal pairs each were used in the experiment. The lists were constructed as follows. In 992 of the $32^2 = 1024$ possible ordered pairs of signals, the first and second signals of the pair are different. These 992 pairs were divided randomly into two groups of 331 pairs each and one group of 330 pairs. Adding the 32 remaining pairs--the pairs in which the first and second signal were identical--to each of these groups brought the total number of signal pairs in two groups, Groups I and II, to 363 and the total number of pairs in Group III to 362. Each of these groups was further trichotomized into an A, B, and a C subset. Subsets IA, IB, IIA, IIB, IIIA, and IIIB each contained 120 pairs, subsets IC and IIC contained 123 pairs each, and subset IIIC contained 122 pairs. The order of presentation for pairs within each subset was determined by a random number table.

With these nine subsets nine lists were then constructed according to the design shown in Table 9. In list 1, the pairs of subset IA are presented first, followed by the pairs of subset IB, and terminated with the pairs of subset IC. Although the same signal pairs appear in the first three lists, the order in which the subsets occur in the lists differs. The order of presentation of pairs within a subset, however, is the same in all lists in which that subset appears.

Each group of Ss responded to one of these nine lists. Since each subset appeared first in one list, second in another list, and third in still another list, each subset (and therefore each of the 992 pairs in which the first and second signals differ) was presented to one group of Ss during the first third of its experimental session, to another group during the middle third of its experimental session, and to still another group during the final third of its session. Because the 32

TABLE 9
The Nine Lists Used in the Experiment

List	First Subset Presented	Second Subset Presented	Third Subset Presented
1	IA	IB	IC
2	IB	IC	IA
3	IC	IA	IB
4	IIA	IIB	IIC
5	IIB	IIC	IIA
6	IIC	IIA	IIB
7	IIIA	IIIB	IIIC
8	IIIB	IIIC	IIIA
9	IIIC	IIIA	IIIB

"identical" pairs appeared in stimulus groups I, II, and III, the "identical" pairs were actually presented to three groups of Ss in each third of the experiment. For example, the pairs in subset IIB were presented to group 5 during the first third, to group 4 during the second third, and to group 6 during the final third of the session. Since every pair was presented during each third of the experiment, it was possible to investigate systematic changes in the patterns of confusion over the course of the experiment.

The nine lists were read as input into a PDP-4 digital computer (manufactured by the Digital Equipment Corporation). The computer then generated the signal pairs in the order specified for each list. The output for each list was recorded from the computer through external analog equipment onto magnetic tape. A temporal unit of .03 sec. was utilized instead of the .05 sec. unit used by Rothkopf, since at the .05 sec. per unit rate Ss confused nonidentical signals only 5% of the time. Since the temporal unit was .03 sec., the duration of dots and 1-unit silences was .03 sec. whereas the duration of dashes and 3-unit silences was .09 sec. As in Rothkopf's experiment, the silent period between the signals of a pair was 1.4 sec. and the interval between pairs was 3.0 sec. A 4.6 sec. silent period was interposed after every 15 pairs during which time Ss were told the item number of the pair which would be presented next. In addition after every 120 pairs Ss were given a one-minute rest.

Preceding the stimuli on each tape were the instructions which appear in the Appendix. The tapes were played on an Ampex 6022 field recorder and were amplified by an Ampex 620 amplifier-speaker system. The speaker was placed on the floor in the front of a 60' x 25' room. The Ss sat in six rows of six Ss each facing the speaker.

The distance from the speaker to the Ss ranged from 12 to 30 feet. The volume was adjusted so as to make the stimuli clearly audible to all Ss, but the sound pressure level was not measured. The total time for each experimental session was 45 minutes--8 minutes for the instructions and practice trials, 32 minutes for stimulus presentation, and 5 minutes for a postexperimental questionnaire.

B. A Restatement of the Hypotheses

As in Rothkopf's experiment, $p(x,y)$ denotes the percentage of "same" responses to the ordered pair (x,y) . The hypotheses of the second model will now be stated in terms of these percentages.

Hypothesis 1.

For all $x, y, z \in \mu_2$, if x is closer to y than to z in the ordering on μ_2 , then $p(x,y) > p(x,z)$,

$$p(x,y) > p(z,x),$$

$$p(y,x) > p(z,x), \text{ and}$$

$$p(y,x) > p(x,z).$$

Hypothesis 2.

For all $x, y, z \in \mu_2$, $(x \neq y \neq z)$, if

(i) y is component-wise between x and z ,

(ii) y is dash-wise between x and z ,

(iii) y is silence-wise between x and z ,

(iv) y is time-wise between x and z , and in addition,

(v) neither $x < y < z$ nor $z < y < x$,

then $p(x,y) > p(x,z)$,

$$p(x,y) > p(z,x),$$

$$p(y,x) > p(z,x), \text{ and}$$

$$p(y,x) > p(x,z).$$

Since the same fundamental considerations motivate both the first and second models, the test of these hypotheses will also test the generality of this facet-theoretic type of approach.

C. Simplexes and Double Simplexes in the Data Matrix

The data from the total experiment, as well as the data obtained during each third of the experiment, can be summarized in a 32×32 matrix whose rows indicate the signal presented first, whose columns indicate the signal presented second, and whose cells indicate the percentage of "same" responses to the corresponding ordered pairs. (These matrices will be shown in the next section of this chapter.) We should again expect each cell on the main diagonal to be greater than any other cell in its row and column, since the diagonal cells indicate the percentage of "same" responses to pairs of identical signals. Some consequences of the hypotheses of the second model for the structure of certain submatrices of the 32×32 data matrix are as follows:

(1) If $w < x < y < z < \dots$, then the submatrix whose rows and columns are ordered w, x, y, z, \dots should be a simplex.

For example, since $(.1.1.) < (.1.3.) < (.1-3.) < (-1-3.) < (-3-3.) < (-3-3-)$ -- $S_1 < S_2 < R_2 < G_2 < G_4 < O_4$ --, it is predicted that the submatrix whose rows and columns are ordered $S_1, S_2, R_2, G_2, G_4, O_4$ will be a simplex.

(2) If (i) $w < x < y < z$ and $w' < x' < y' < z'$,

(ii) w, x, y , and z have the same number of dashes and the same number of long silences as w', x', y' , and z' , respectively, and

(iii) w, x, y , and z differ from w', x', y' , and z' , respectively, on exactly the same components,

then the submatrix whose rows and columns are ordered $w, x, y, z, w', x', y', z'$ should be a double simplex.

Example:

Let $w = (-1.1.)$, $x = (-1.1-)$, $y = (-1.3-)$, $z = (-3.3-)$,

$w' = (.1-1.)$, $x' = (.1-1-)$, $y' = (.1-3-)$ and $z' = (.3-3-)$.

Since (i) $w < x < y < z$ and $w' < x' < y' < z'$,

(ii) $d(w) = d(w')$ and $s(w) = w(w')$,

$d(x) = d(x')$ and $s(x) = s(x')$,

$d(y) = d(y')$ and $s(y) = s(y')$,

$d(z) = d(z')$ and $s(z) = s(z')$, and

(iii) w, x, y , and z differ from w', x', y' , and z' ,

respectively, only on the first and third components,

it is predicted that the submatrix whose rows and columns are ordered $(-1.1.)$, $(-1.1-)$, $(-1.3-)$, $(-3.3-)$, $(.1-1.)$, $(.1-1-)$, $(.1-3-)$, $(.3-3-)$ -- $D_1, K_1, K_2, K_4, R_1, W_1, W_2, W_4$ -- will be a double simplex.

IV. Results and Discussion

A. The Hypotheses

The percentage of "same" responses to each ordered pair in the total experiment is given in Tables 10, 11, and 12. These three permutations have been provided in order to make it easier to comprehend the patterns of confusion among the signals. As before, the rows indicate the signal presented first, and the columns indicate the signal presented second.

The mean percentage of "same" judgments for the pairs associated with the diagonal cells, 95.4%, is greater than the corresponding mean in Rothkopf's data, 89.3%. The same holds true for the pairs associated with the off-diagonal cells; the mean percentage of "same" responses to

Table 10 Percentage of "same" judgments obtained for all ordered pairs of the 32 Morse-like rhythmic patterns. Signals grouped according to dot-and-dash pattern.

	s ₁	s ₂	s ₃	s ₄	v ₁	v ₂	v ₃	v ₄	p ₁	p ₂	p ₃	p ₄	v ₁	v ₂	v ₃	v ₄	k ₁	k ₂	k ₃	k ₄	c ₁	c ₂	c ₃	c ₄	o ₁	o ₂	o ₃	o ₄																				
-1-1-s ₁	97	79	44	02	94	18	11	02	81	28	05	00	64	11	01	00	16	01	00	00	31	04	02	01	06	02	01	03	01	02	02																	
-1-1-s ₂	44	94	60	25	54	04	45	11	02	62	27	03	56	54	39	17	32	38	10	00	41	22	21	00	19	07	06	02	04	00	01	02																
-3-1-s ₃	42	69	94	47	38	53	91	13	30	11	59	12	46	33	54	27	23	20	16	02	22	03	45	03	19	05	03	02	06	01	03	00																
-3-3-s ₄	02	28	44	96	05	17	55	78	04	15	57	59	02	28	19	47	06	07	26	00	05	07	12	35	06	23	19	36	12	06	15	03																
-1-1-u ₁	63	57	44	03	98	61	35	00	06	49	06	03	74	21	00	03	91	13	04	05	73	00	07	01	23	06	03	00	16	00	01	02																
-1-3-u ₂	03	63	30	20	39	94	45	27	44	39	22	03	15	52	34	00	66	77	21	04	61	48	34	15	29	14	00	05	19	07	14	03																
-3-1-u ₃	04	39	56	17	09	29	94	34	30	14	53	19	18	21	57	15	42	38	64	10	32	22	67	16	35	05	23	02	29	05	12	03																
-3-3-u ₄	00	04	04	45	01	14	52	97	04	20	50	55	06	13	12	26	12	19	59	67	17	29	26	64	22	35	19	13	26	26	44	13																
-1-1-r ₁	27	42	23	07	73	52	22	07	94	64	23	12	30	49	22	09	72	30	14	01	62	11	12	05	51	16	12	02	22	00	04	00																
-1-3-r ₂	05	31	19	10	16	38	25	14	46	95	33	27	15	47	34	32	59	06	33	11	19	36	19	02	45	53	21	06	26	12	07	01																
-3-1-r ₃	05	10	29	27	05	17	53	37	43	40	94	05	00	31	52	27	22	28	04	19	24	31	43	30	38	24	62	11	20	05	55	03																
-3-3-r ₄	02	03	04	31	01	06	17	29	10	49	66	94	02	23	21	46	16	21	45	61	07	17	06	31	20	39	44	61	17	30	34	05																
-1-1-b ₁	47	43	28	03	75	13	16	02	69	40	17	00	96	34	50	07	33	11	00	03	94	26	05	02	67	19	03	04	00	02	06	00																
-1-3-b ₂	06	47	09	19	14	39	18	17	26	43	34	19	46	94	42	51	26	35	15	05	51	79	37	34	60	73	33	02	43	35	16	00																
-3-1-b ₃	01	19	30	06	04	36	36	05	06	21	35	07	26	51	94	00	00	06	16	01	47	19	97	15	31	21	47	05	05	04	20	01																
-3-3-b ₄	01	01	03	26	03	05	25	42	07	07	16	30	06	28	67	97	13	06	25	31	05	19	52	03	26	22	76	00	35	25	67	03																
-1-1-w ₁	03	20	22	09	40	51	33	19	64	73	25	19	42	40	17	11	94	69	31	06	76	49	10	00	68	54	12	06	70	31	00	00																
-1-3-w ₂	02	07	05	04	14	54	21	22	24	69	19	06	00	31	19	13	62	94	44	14	39	64	29	19	27	56	19	03	59	41	34	11																
-3-1-w ₃	01	04	14	15	02	16	51	60	15	21	72	41	02	00	19	44	32	36	94	51	12	29	31	49	37	53	56	29	50	34	76	31																
-3-3-w ₄	00	01	03	25	02	03	05	46	03	14	19	55	02	06	00	50	06	36	56	93	06	09	12	59	22	42	36	65	33	54	69	74																
-1-1-k ₁	07	19	08	03	44	37	27	06	33	40	32	05	69	56	47	16	69	26	17	06	95	74	46	06	70	27	11	01	67	16	00	01																
-1-3-k ₂	01	14	03	01	03	19	18	06	05	23	14	06	07	60	25	31	22	40	14	15	54	96	33	37	37	47	41	10	56	72	37	07																
-3-1-k ₃	02	02	17	03	04	26	56	09	06	13	13	00	11	22	00	46	14	13	13	06	61	28	97	67	18	17	79	12	27	09	06	02																
-3-3-k ₄	01	01	04	06	01	05	08	42	00	02	06	15	02	10	28	81	02	11	15	16	07	27	75	98	05	26	56	37	32	34	83	53																
-1-1-c ₁	01	06	04	10	08	14	11	18	49	48	47	19	50	80	37	38	78	23	40	13	78	53	27	14	93	90	22	11	82	33	36	06																
-1-3-c ₂	01	06	01	05	75	07	10	11	10	46	27	32	00	57	00	54	30	44	36	21	28	42	23	44	71	93	59	57	73	71	50	19																
-3-1-c ₃	01	01	05	04	77	05	11	12	03	04	56	22	05	17	60	60	07	13	56	21	29	16	57	45	43	37	06	52	14	42	00	17																
-3-3-c ₄	01	02	00	05	01	03	03	22	01	07	11	35	01	01	07	62	06	00	27	33	06	11	00	55	26	59	74	97	18	47	00	51																
-1-1-o ₁	02	00	05	11	10	04	11	31	19	12	25	00	04	16	09	33	31	20	38	31	51	50	31	28	76	82	23	14	97	82	57	00																
-1-3-o ₂	00	00	02	02	01	04	06	15	04	07	07	11	03	24	03	31	21	55	19	47	08	56	14	41	35	56	25	51	80	94	00	56																
-3-1-o ₃	00	02	01	02	00	03	10	15	03	09	36	14	00	01	28	49	08	17	29	27	18	22	59	73	19	20	74	60	64	52	97	56																
-3-3-o ₄	00	02	03	06	02	01	01	09	00	00	00	17	01	02	01	05	00	06	06	42	01	05	03	53	07	35	26	75	09	64	00	97																
s ₁ s ₂ s ₃ s ₄	v ₁	v ₂	v ₃	v ₄	p ₁	p ₂	p ₃	p ₄	v ₁	v ₂	v ₃	v ₄	k ₁	k ₂	k ₃	k ₄	c ₁	c ₂	c ₃	c ₄	o ₁	o ₂	o ₃	o ₄	s ₁	s ₂	s ₃	s ₄	v ₁	v ₂	v ₃	v ₄	p ₁	p ₂	p ₃	p ₄	k ₁	k ₂	k ₃	k ₄	c ₁	c ₂	c ₃	c ₄	o ₁	o ₂	o ₃	o ₄

Table 11 Percentage of "same" judgments obtained for all ordered pairs of the 32 Morse-like rhythmic patterns. Signals grouped according to silence patterns.

	S ₁	U ₁	R ₁	D ₁	W ₁	K ₁	G ₁	O ₁	S ₂	U ₂	R ₂	D ₂	W ₂	K ₂	G ₂	O ₂	S ₃	U ₃	R ₃	D ₃	W ₃	K ₃	G ₃	O ₃	S ₄	U ₄	R ₄	D ₄	W ₄	K ₄	G ₄	O ₄	
•1-1- S ₁	97	94	81	66	16	31	06	03	79	18	28	11	01	04	02	01	44	11	05	01	00	02	01	02	02	02	00	00	00	01	03	02	S ₁
•1-1- U ₁	63	98	86	74	91	73	23	16	57	61	49	21	13	09	06	00	44	35	06	08	04	07	03	01	03	00	03	03	05	01	00	02	U ₁
•1-1- R ₁	27	73	94	50	72	62	51	22	42	52	64	49	30	11	16	00	23	22	33	22	14	12	12	04	07	07	12	09	01	05	02	09	R ₁
•1-1- D ₁	47	75	69	96	33	94	67	09	43	13	40	34	11	26	19	02	28	16	17	50	00	05	03	06	03	02	00	07	03	02	04	00	D ₁
•1-1- W ₁	03	40	64	42	94	76	68	70	20	51	73	40	69	49	54	31	22	33	25	17	31	18	12	08	09	19	19	11	06	09	06	00	W ₁
•1-1- K ₁	07	44	33	69	69	95	70	67	19	37	40	56	26	74	27	16	08	27	32	47	17	46	11	08	03	06	05	16	06	06	01	01	K ₁
•1-1- G ₁	01	08	49	50	78	78	93	82	08	14	48	80	23	53	90	33	04	11	47	37	40	27	22	38	10	18	19	38	13	14	11	06	G ₁
•1-1- O ₁	02	10	19	04	31	51	76	97	00	04	12	16	20	50	82	82	05	11	25	09	38	31	23	57	11	31	08	33	31	28	14	09	O ₁
•1-3- S ₂	44	54	62	56	32	41	19	04	94	84	62	54	38	22	07	00	69	45	27	39	10	21	06	01	25	11	03	17	00	00	02	02	S ₂
•1-3- U ₂	03	39	44	15	66	61	29	19	63	94	39	52	77	48	14	07	30	45	22	24	21	34	09	14	20	27	03	08	04	15	05	03	U ₂
•1-3- R ₂	05	16	48	15	59	19	45	28	31	38	95	47	86	36	53	12	19	25	33	24	33	19	21	07	10	14	27	32	11	02	06	01	R ₂
•1-3- D ₂	06	14	28	46	26	51	69	43	47	39	43	94	35	79	73	35	09	18	34	42	15	37	33	16	19	17	19	51	05	34	09	00	D ₂
•1-3- W ₂	02	14	24	09	62	39	27	59	07	54	69	37	94	64	56	41	05	21	19	19	44	29	19	34	04	22	06	13	14	19	03	11	W ₂
•1-3- K ₂	01	03	05	07	22	54	37	56	14	19	23	68	48	98	47	72	03	18	14	25	14	33	41	37	01	06	06	31	15	37	18	07	K ₂
•1-3- G ₂	01	05	10	08	30	28	71	73	08	07	46	57	44	42	93	71	01	10	27	08	36	23	59	50	05	11	32	54	21	44	57	19	G ₂
•1-3- O ₂	00	01	04	03	21	08	35	80	00	04	07	24	55	58	94	94	02	06	07	03	19	14	25	60	02	15	11	31	47	41	57	56	O ₂
•3-1- S ₃	42	38	50	48	23	22	19	06	69	53	31	33	09	03	05	01	94	91	59	54	16	45	03	05	47	13	27	02	03	02	00	00	S ₃
•3-1- U ₃	04	09	30	18	42	52	35	29	30	29	14	21	30	22	05	05	56	94	53	57	64	67	23	12	17	34	19	15	18	16	02	03	U ₃
•3-1- R ₃	05	05	43	08	22	24	38	20	10	17	40	31	28	31	24	05	29	53	94	52	84	43	62	55	27	37	65	27	19	30	11	03	R ₃
•3-1- D ₃	01	04	06	26	09	47	31	05	19	36	21	51	06	19	21	04	30	36	35	94	16	97	47	20	16	05	07	60	01	15	05	01	D ₃
•3-1- W ₃	01	02	15	02	32	12	37	50	04	16	21	08	36	29	53	34	14	51	72	19	94	31	56	76	15	60	41	44	51	49	29	31	W ₃
•3-1- K ₃	02	04	06	11	14	61	18	27	02	26	13	22	13	28	17	09	17	56	13	80	13	97	79	68	03	09	08	46	06	67	12	02	K ₃
•3-1- G ₃	01	07	03	05	07	29	43	14	01	05	04	27	13	16	37	42	05	11	56	68	58	57	96	89	04	12	22	60	21	45	52	17	G ₃
•3-1- O ₃	00	00	03	00	08	18	19	64	02	03	09	07	17	22	20	52	01	10	36	28	29	50	74	97	02	15	14	49	27	73	69	56	O ₃
•3-3- S ₄	02	05	04	02	06	05	06	12	28	17	15	28	07	07	23	08	44	55	57	19	28	12	19	15	96	78	59	47	08	35	36	03	S ₄
•3-3- U ₄	00	01	04	06	12	17	22	26	04	14	20	13	19	29	35	26	04	52	50	12	59	26	19	44	45	97	55	28	67	64	13	13	U ₄
•3-3- R ₄	02	01	10	02	16	07	20	17	03	06	49	23	21	17	39	30	04	17	66	21	45	06	44	54	31	29	94	48	61	31	61	05	R ₄
•3-3- D ₄	01	03	07	06	13	05	28	33	01	05	07	28	06	19	22	25	03	25	16	67	25	52	78	67	26	42	30	97	31	83	60	03	D ₄
•3-3- W ₄	00	02	03	02	06	06	22	33	01	03	14	06	38	09	42	54	03	05	19	08	56	12	38	69	25	46	55	50	93	59	65	14	W ₄
•3-3- K ₄	01	01	00	02	02	07	05	32	01	05	02	10	11	27	28	34	04	08	06	28	15	75	56	83	06	42	15	81	16	98	37	53	K ₄
•3-3- G ₄	01	01	01	01	06	06	26	18	02	03	07	01	08	11	59	47	00	03	11	07	27	08	74	80	05	22	35	62	33	55	97	51	G ₄
•3-3- O ₄	00	02	00	01	00	01	07	09	02	01	00	02	06	05	35	64	03	01	00	01	06	03	26	89	06	09	17	05	42	53	75	97	O ₄
	S ₁	U ₁	R ₁	D ₁	W ₁	K ₁	G ₁	O ₁	S ₂	U ₂	R ₂	D ₂	W ₂	K ₂	G ₂	O ₂	S ₃	U ₃	R ₃	D ₃	W ₃	K ₃	G ₃	O ₃	S ₄	U ₄	R ₄	D ₄	W ₄	K ₄	G ₄	O ₄	

Table 12 Percentage of "Same" Judgments Obtained for All Ordered Pairs of the 32 Morse-like Rhythmic Patterns. Signals Grouped According to Temporal Length.

	S ₁	S ₂	S ₃	S ₄	U ₁	R ₁	D ₁	S ₄	U ₃	R ₃	D ₃	U ₂	R ₂	D ₂	W ₁	K ₁	C ₁	U ₄	R ₄	D ₄	W ₃	K ₃	C ₃	W ₂	K ₂	C ₂	O ₁	W ₄	K ₄	C ₄	O ₂	O ₄	S ₁	
-1-1-S ₁	97	44	79	94	81	66	02	11	05	01	18	28	11	16	31	06	02	00	00	00	00	02	01	01	04	02	03	00	01	03	02	01	02	S ₁
-3-1-S ₃	42	94	69	38	50	48	47	91	59	54	53	31	33	23	22	19	13	12	27	16	45	03	09	03	05	06	02	03	02	05	01	00	S ₃	
-1-3-S ₂	44	69	94	54	62	56	25	45	27	39	84	62	54	32	41	19	11	03	17	10	21	06	38	22	07	04	00	00	02	01	00	02	S ₂	
-1-1-U ₁	63	44	57	98	86	74	03	35	06	08	61	49	21	91	73	23	00	03	03	04	07	03	13	09	06	16	05	01	00	01	00	02	U ₁	
-1-1-R ₁	27	2	42	73	94	50	07	22	33	22	52	64	49	72	62	51	07	12	09	14	12	12	30	11	16	22	01	05	02	04	00	09	R ₁	
-1-1-D ₁	47	28	43	75	69	96	03	16	17	50	13	40	34	33	94	67	02	00	07	00	05	03	11	26	19	09	03	02	04	06	02	00	D ₁	
-3-3-S ₄	02	44	28	05	04	02	96	55	57	19	17	15	28	06	05	06	78	59	47	28	12	19	07	07	23	12	08	35	36	15	08	03	S ₄	
-3-1-U ₃	04	56	30	09	30	18	17	94	53	57	29	14	21	42	52	35	34	19	15	64	67	23	30	22	05	29	18	16	02	12	05	03	U ₃	
-3-1-R ₃	05	29	10	05	43	08	27	53	94	52	17	40	31	22	24	38	37	65	27	84	43	62	28	31	24	20	19	30	11	55	05	03	R ₃	
-3-1-D ₃	01	30	19	04	06	26	06	36	35	94	36	21	51	09	47	31	05	07	60	16	97	47	06	19	21	05	01	15	05	20	04	01	D ₃	
-1-3-U ₂	03	30	63	39	44	15	20	45	22	24	94	39	52	66	61	29	26	03	08	21	34	09	77	48	14	19	04	15	05	14	07	03	U ₂	
-1-3-R ₂	05	19	3	16	48	15	10	25	33	24	38	95	47	59	19	45	14	27	32	33	19	21	86	36	53	28	11	02	06	07	12	01	R ₂	
-1-3-D ₂	06	09	47	14	28	46	19	18	34	42	39	43	94	26	51	69	17	19	51	15	37	33	35	79	73	43	05	34	09	16	35	00	D ₂	
-1-1-W ₁	03	22	20	40	64	42	09	33	25	17	51	73	40	94	76	68	19	19	11	31	18	12	69	49	54	70	06	09	06	08	31	00	W ₁	
-1-1-K ₁	07	08	19	44	33	69	03	27	32	47	37	40	56	69	95	70	06	05	16	17	46	11	26	74	27	67	06	06	01	08	16	01	K ₁	
-1-1-C ₁	01	04	08	08	49	50	10	11	7	37	14	48	80	78	78	93	18	19	38	40	27	22	23	53	90	82	13	14	11	38	33	06	C ₁	
-3-3-U ₄	00	04	04	01	04	06	45	52	50	12	14	20	13	12	17	22	97	55	28	59	26	19	19	29	35	26	67	64	13	44	26	13	U ₄	
-3-3-R ₄	02	04	03	01	10	02	31	17	66	21	06	49	23	16	07	20	29	94	48	45	06	44	21	17	39	17	61	31	61	54	30	05	R ₄	
-3-3-D ₄	01	03	01	03	07	06	26	25	16	67	05	07	28	13	05	28	42	30	97	25	52	78	06	19	22	33	31	83	60	67	25	03	D ₄	
3-1-W ₃	01	14	04	02	15	02	15	51	72	19	16	21	08	32	12	37	60	41	44	94	31	56	36	29	53	50	51	49	29	76	34	31	W ₃	
-3-1-K ₃	02	17	02	04	06	11	03	56	13	80	26	13	22	14	61	18	09	08	46	13	97	79	13	28	17	27	06	67	12	68	09	02	K ₃	
-3-1-C ₃	01	05	01	07	03	05	04	11	56	68	05	04	27	07	29	43	12	22	60	58	57	96	13	16	37	14	21	45	52	89	42	17	C ₃	
-1-3-W ₂	02	05	07	14	24	09	04	21	19	19	54	69	37	62	39	27	22	06	13	44	29	19	94	64	56	59	14	19	03	34	41	11	W ₂	
-1-3-K ₂	01	03	14	03	05	07	01	18	14	25	19	23	68	22	54	37	06	06	31	14	33	41	48	98	47	56	15	37	18	37	72	07	K ₂	
-1-3-C ₂	01	01	08	05	10	08	05	10	27	08	07	46	57	30	28	71	11	32	54	46	23	59	44	42	93	73	21	44	57	50	71	19	C ₂	
-1-1-O ₁	02	05	00	10	19	04	11	11	25	09	04	12	16	31	1	76	31	08	33	38	31	23	20	50	82	97	31	28	14	57	82	09	O ₁	
-3-3-W ₄	00	03	01	02	03	02	25	05	19	08	03	14	06	06	06	22	46	55	50	56	12	38	38	09	42	33	93	59	65	69	54	74	W ₄	
-3-3-K ₄	01	04	01	01	00	02	06	08	06	28	05	02	10	02	07	05	42	15	81	15	75	56	11	27	28	32	16	98	37	83	34	53	K ₄	
-3-3-C ₄	01	00	02	01	01	01	05	03	11	07	03	07	01	06	06	26	22	35	62	27	08	74	08	11	59	18	33	55	97	80	47	51	C ₄	
-3-1-O ₃	00	01	02	00	03	00	02	10	36	28	03	09	07	08	18	19	15	14	49	29	50	74	17	22	20	64	27	73	69	97	52	56	O ₃	
-1-3-O ₂	00	02	00	01	04	03	02	06	07	03	04	07	24	21	08	35	15	11	31	19	14	25	55	58	58	80	47	41	51	60	94	56	O ₂	
-3-3-O ₄	00	03	02	02	00	01	06	01	00	01	01	01	00	02	00	01	09	17	05	06	03	26	06	05	35	09	42	53	75	89	64	97	O ₄	
S ₁ S ₂ S ₃ U ₁ R ₁ D ₁ S ₄ U ₃ R ₃ D ₃ U ₂ R ₂ D ₂ W ₁ K ₁ C ₁ U ₄ R ₄ D ₄ W ₃ K ₃ C ₃ W ₂ K ₂ C ₂ O ₁ W ₄ K ₄ C ₄ O ₃ O ₂ O ₄																																		

the objectively different pairs is 26% in the current data, whereas it is only 17.6% in the Morse Code data. The difference in the confusion rates for the two experiments is probably a function of the difference in the populations, the difference in the signals, and the difference in the speeds at which the signals were presented. Since we have predicted only the relative magnitude of the percentages, the overall difference in confusion rate does not directly affect the hypotheses.

One can observe in either Table 10, 11, or 12 that in only two instances is a diagonal cell equalled or exceeded by another cell in its row or column-- $p(D_3, D_3) < p(D_3, K_3)$, and $p(K_3, K_3) = p(D_3, K_3)$.

As before, the predictions following from the first hypothesis will be divided into two classes. In the first class are those predictions in which y is immediately bounded by x and z . Of the 1280 predictions in this class, 1262 (99%) are correct. The predictions in which y is not immediately bounded by x and z are in the second class. Of the 3120 predictions in the second class, 2953 (95%) are correct. Although 167 of the predictions are incorrect, only 8 of these errors of prediction involve a difference greater than 2%. In total, 4125 of the 4400 predictions (96%) following from the first hypothesis are correct.

One might have expected hypothesis 1 to fail in cases where x , y , and z have the same dot-and-dash pattern. For example, since the tones of O_1 , (-1-1-), and O_4 , (-3-3-), are evenly spaced, whereas the tones of O_2 , (-1-3-) are unevenly spaced, one might have predicted that O_1 would be confused more often with O_4 than with O_2 . The data, however, support the hypothesis in this instance and in all other instances in which x , y , and z have the same dot-and-dash pattern.

Let us now consider predictions in which x, y, and z have the same pattern of silent intervals. Since there are four possible patterns of silences, the signals can be divided into 4 subsets on the basis of their patterns of silences. Subset 1 contains the 8 signals whose alphabetic equivalents have 1's as subscripts, i.e., the 8 signals which are also in the IMC. Subsets 2, 3, and 4 contain the signals whose alphabetic equivalents have 2's, 3's, and 4's, respectively, as subscripts.

Hypothesis 1 leads to 144 predictions among the signals of each subset. The number of correct predictions for subsets 1, 2, 3, and 4, respectively, are 141, 142, 139, and 136. Furthermore, the number of correct predictions among the signals of subset 1, 141, is quite close to the number of correct predictions among these signals in Rothkopf's study, 140. Thus the validity of hypothesis 1 is not affected by the difference in the particular pattern of silent intervals nor by the context of other alternatives.

Two matrices that have been predicted to be simplexes appear in Table 13. Since the predictions relevant to the first hypothesis are almost always correct, there are very few violations of the simplex patterns.

Having shown that the data strongly support hypothesis 1, let us now focus our attention on hypothesis 2. Of the 7712 predictions following directly from the second hypothesis, 7118 (91%) are in accord with the data. Again, however, very few of the errors of prediction involve differences greater than 2%.

For each of the four subsets described above, hypothesis 2 leads to 96 predictions of relative confusability. The number of correct predictions among the signals of subset 1, 92, is only slightly

TABLE 13

Two Approximate Simplexes in the Total Data Matrix

	S_1	S_2	R_2	G_2	G_4	O_4
• 1 • 1 • S_1	97	79	28	02	03	02
• 1 • 3 • S_2	44	94	62	07	02	02
• 1 - 3 • R_2	05	31	95	53	06	01
- 1 - 3 • G_2	01	08	46	93	57	19
- 3 - 3 • G_4	01	02	07	59	97	51
- 3 - 3 - O_4	00	02	00	35	75	97

	S_1	D_1	K_1	K_3	K_4	O_4
• 1 • 1 • S_1	97	66	31	02	01	02
- 1 • 1 • D_1	47	96	94	05	02	00
- 1 • 1 - K_1	07	69	95	46	06	01
- 1 • 3 - K_3	02	11	61	97	67	02
- 3 • 3 - K_4	01	02	07	75	98	53
- 3 - 3 - O_4	00	01	01	03	53	97

greater than the number of correct predictions for these signals in the Morse Code context, 88. Thus, although the percentage of "same" judgment in the current experiment differs considerably from the percentages obtained by Rothkopf for these pairs, the relative magnitude of the percentages is quite similar in the two experiments. The number of correct predictions for subsets 2, 3, and 4 are 94, 91, and 83, respectively.

Violations of the second hypothesis are most likely to occur when x, y, and z have the same temporal length. In these cases 674 of the 768 predictions (88%) are correct.

Table 14 shows two matrices which have been predicted to be double simplexes. We can observe that the four 4 x 4 submatrices of each matrix are approximately simplexes and that the values in the cells of submatrix 1 and 4 are generally greater than the values in the corresponding cells of submatrices 2 and 3. Thus, both matrices in Table 14 are approximately double simplexes.

B. Other Patterns of Confusion Among the Morse-like Rhythmic Patterns

As in Rothkopf's experiment, signals that differ on a single component (particularly the final component) are confused with each other more often than signals differing in any other way. The mean confusion rate for the 160 ordered pairs of this type is 56%. A breakdown of the average confusion rate for differences on each of the 5 components is as follows:

Differ on only the fifth component	-- 74%
Differ on only the fourth component	-- 61%
Differ on only the third component	-- 57%
Differ on only the first component	-- 50%
Differ on only the second component	-- 37%

TABLE 14

Two Approximate Double Simplexes in the Total Data Matrix

	D ₁	K ₁	K ₂	K ₄	R ₁	W ₁	W ₂	W ₄
-1-1- D ₁	96	94	26	02	69	33	11	03
-1-1- K ₁	69	95	74	06	33	69	26	06
-1-3- K ₂	07	54	98	37	05	22	48	15
-3-3- K ₄	02	07	27	98	00	02	11	16
				1\2				
				3\4				
•1-1- R ₁	50	62	11	05	94	72	30	01
•1-1- W ₁	42	76	49	09	64	94	69	06
•1-3- W ₂	09	39	64	19	24	62	94	14
•3-3- W ₄	02	06	09	59	03	06	38	93
	S ₃	R ₃	W ₃	O ₃	S ₂	R ₂	W ₂	O ₂
•3-1- S ₃	94	59	16	05	69	31	09	01
•3-1- R ₃	29	94	84	55	10	40	28	05
•3-1- W ₃	14	72	94	76	04	21	36	34
-3-1- O ₃	01	36	29	56	02	09	17	52
				1\2				
				3\4				
•1-3- S ₂	69	27	10	01	94	62	38	00
•1-3- R ₂	19	33	33	07	31	95	86	12
•1-3- W ₂	05	19	44	34	07	69	94	41
-1-3- O ₂	02	07	19	60	00	07	55	94

Observe that on the average there are more "same" responses when signals differ on the final component than when they differ on any other single component. Furthermore, there are more confusions when signals differ on the second tone (the third component) than when they differ on the first tone (the first component); and there are also more confusions when signals differ on the second silent interval (the fourth component) than on the first silent interval (the second component). It is somewhat surprising to observe that differences on the second component, a silence, are easier to detect than differences on any other component. This demonstrates the important role of the silence, or rhythmic, pattern in the perceptual process.

The order in which the signals of certain pairs are presented has a considerable effect on the probability of confusing the signals of these pairs. If $x < y$ and in addition $d(x) < d(y)$, then x and y are much more likely to be confused if x is presented before y than if y is presented before x , particularly if y is not more than 4 units (in temporal length) longer than x . If $x < y$, $d(x) < d(y)$, and y is two units longer than x , then $p(x,y) > p(y,x)$ 43 times, $p(x,y) = p(y,x)$ 1 time, and $p(x,y) < p(y,x)$ 4 times. The mean of these $p(x,y)$'s is 69%, whereas the mean of these $p(y,x)$'s is 52%. If $x < y$, $d(x) < d(y)$, and y is 4 units longer than x , then $p(x,y) > p(y,x)$ 49 times, $p(x,y) = p(y,x)$ 6 times, and $p(x,y) < p(y,x)$ 17 times. The mean of these $p(x,y)$'s is 17%, whereas the mean of these $p(y,x)$'s is 10%.

It is interesting to note that if $x < y$ and $d(x) = d(y)$, then the mean probability of confusing x and y is 41%, irrespective of the order in which x and y are presented. Thus, although there is a positive "time-error" associated with the duration of the tones of signals, there is no such effect associated with the duration of silent intervals of signals.

The next most common kind of misperception is the confusion of signals that have the same combination of components, but differ in terms of an inversion of a pair of tones or a pair of silences. Although the signals of each pair in this category always have the same temporal length and differ on two components, the converse is not true. In order for a pair of signals to be in this category, the signals must differ on either the first and third, the first and fifth, the third and fifth, or the second and fourth components. For example (-1.3-, .1-3-), (.3.3-, -3.3.), (.1-1., .1.1-) and (.1.3., .3.1.) are in this category. The average percentage of "same" judgments for the 64 ordered pairs in this category is 50%. The percentages corresponding to each of the four subcategories is as follows:

- Inversion of third and fifth components -- 56%,
- Inversion of first and fifth components -- 54%,
- Inversion of first and third components -- 46%,
- Inversion of second and fourth components -- 45%.

We can see that the later in the signal the inversion of a pair of tones occurs, the higher the likelihood that Ss will fail to notice that the signals differ. We can also observe that Ss detect a difference in the order of the silences slightly better than a difference in the order of the tones.

The third most common kind of confusion involves pairs of signals which have the same temporal length, but which differ on one tone and one silence. The average percentage of confusions for pairs of signals in this category is 42%. The breakdown of the percentages into the six subcategories is as follows:

Signals differ on the third and fourth components -- 61%,
Signals differ on the fourth and fifth components -- 57%,
Signals differ on the first and fourth components -- 37%,
Signals differ on the first and second components -- 36%,
Signals differ on the second and third components -- 31%,
Signals differ on the second and fifth components -- 29%.

Before concluding this section I shall point out some "interactions" between the tones and silences of signals.

(1) Ss confuse signals differing on a single component, a silence, more often if a dash, rather than a dot, precedes the silence on which they differ.

(2) Ss confuse signals differing on a single component, a silence, more often if a dash rather than a dot follows the silence on which they differ.

(3) Ss confuse signals differing on a single component, a tone, more often if a short silence rather than a long silence precedes the tone on which they differ.

C. Effects of Practice on the Perception of the Signals

In the experimental design it was pointed out that since each pair of signals was presented to one group during the first third of its experimental session, to another group during the middle third of its experimental session, and to still another group during the final third of its experimental session, systematic changes in the patterns of confusion over the course of the experiment could be investigated. Tables 15, 16, and 17 show the percentage of "same" responses to all signal pairs during the first, middle, and final thirds of the experiment, respectively. In other words, the percentages in Tables 15 - 17

Table 15 The percentage of "same" judgments obtained during the first third of each experimental session for the 32 Morse-like rhythmic patterns.

	S ₁	S ₂	S ₃	S ₄	U ₁	U ₂	U ₃	U ₄	R ₁	R ₂	R ₃	R ₄	D ₁	D ₂	D ₃	D ₄	W ₁	W ₂	W ₃	W ₄	K ₁	K ₂	K ₃	K ₄	C ₁	C ₂	C ₃	C ₄	O ₁	O ₂	O ₃	O ₄	
-1-1-	100	83	61	05	94	19	14	03	83	33	08	00	72	17	00	00	14	03	00	00	33	00	06	00	08	03	03	00	00	00	00	00	03
-1-3-	42	94	86	42	67	86	69	17	69	56	35	03	78	67	58	22	44	56	17	44	47	31	33	00	31	06	03	03	00	00	00	00	03
-3-1-	47	78	97	50	56	64	89	08	53	44	67	17	67	31	50	42	28	22	06	03	25	06	69	03	31	03	06	03	08	00	00	00	00
-3-3-	00	25	25	94	11	19	61	83	08	25	72	75	00	42	19	42	11	14	44	08	00	06	42	39	17	33	19	42	08	19	25	03	03
-1-1-	75	72	53	08	97	47	47	00	89	69	11	00	81	22	08	03	97	08	00	00	81	11	08	03	42	06	00	00	17	00	03	03	03
-1-3-	00	67	44	25	44	92	72	14	50	42	13	03	26	53	42	08	81	75	22	03	75	61	47	11	44	22	08	00	36	11	25	00	00
-3-1-	03	33	56	17	14	17	97	36	50	19	11	28	17	33	58	19	53	36	75	22	64	28	83	14	44	08	19	00	36	06	14	00	00
-3-3-	00	03	06	67	00	17	56	100	03	22	53	61	02	19	14	22	28	53	69	61	19	28	33	06	19	56	22	17	47	28	64	08	08
-1-1-	42	56	22	08	86	53	33	11	94	61	36	22	58	64	36	03	69	47	22	00	78	14	17	06	53	25	14	00	19	00	06	14	00
-1-3-	03	47	28	19	28	53	31	25	47	97	31	36	19	47	28	28	56	97	47	19	28	42	28	00	50	58	31	06	61	00	11	00	00
-3-1-	06	19	50	36	03	17	11	47	42	56	94	69	03	53	69	42	33	39	81	08	36	42	33	28	44	44	72	11	33	06	72	03	03
-3-3-	03	03	03	50	03	03	11	39	08	56	81	97	00	28	39	69	31	25	58	58	17	31	11	39	25	50	50	53	25	31	56	00	00
-1-1-	44	53	50	00	83	14	25	03	83	11	31	00	97	47	58	11	36	19	00	03	94	31	25	00	69	14	00	03	08	00	03	00	00
-1-3-	14	42	06	22	25	50	19	19	33	61	53	28	75	97	50	50	33	58	28	06	61	67	44	47	75	89	31	11	58	47	19	00	00
-3-1-	00	44	36	11	08	33	61	06	08	33	36	19	28	61	97	72	08	11	22	03	67	25	94	14	42	25	33	06	03	03	14	00	00
-3-3-	00	00	00	47	03	06	31	44	19	06	28	31	11	22	64	97	22	08	39	44	03	28	67	94	56	25	81	56	53	42	75	06	06
-1-1-	00	17	31	14	44	42	39	22	72	72	44	28	42	50	11	11	97	69	39	06	94	58	19	08	86	69	14	03	58	44	14	00	00
-1-3-	00	14	06	03	17	56	31	22	31	75	19	06	08	47	28	11	47	94	50	14	31	72	44	19	36	69	17	03	67	36	56	11	00
-3-1-	00	03	19	25	03	28	61	81	17	28	69	61	06	17	19	56	56	47	92	33	22	47	39	75	53	50	72	33	61	42	78	28	00
-3-3-	00	00	00	22	00	03	08	58	06	14	19	58	00	03	08	58	08	33	47	94	08	08	11	42	31	58	56	72	42	67	86	75	00
-1-1-	08	25	14	03	61	47	36	08	42	50	42	03	78	72	47	22	75	28	17	08	97	81	44	06	89	58	22	03	72	22	06	00	00
-1-3-	00	14	06	03	03	31	28	14	06	28	17	11	03	78	19	44	25	61	19	17	64	100	36	50	39	69	22	19	69	81	44	06	06
-3-1-	03	06	22	03	00	39	72	14	11	14	17	08	14	36	81	58	22	17	11	14	78	39	97	72	25	39	81	14	36	08	56	03	03
-3-3-	03	00	03	06	00	00	14	14	00	03	06	13	03	11	22	94	03	17	25	19	06	39	86	100	06	33	58	36	58	47	89	53	03
-1-1-	03	06	08	17	14	17	11	22	56	42	50	25	53	89	42	50	89	36	44	25	75	58	28	06	97	97	33	08	92	42	64	00	00
-1-3-	00	06	00	03	03	11	19	14	17	25	31	33	11	61	17	72	36	44	53	31	39	50	31	69	86	94	72	72	89	69	58	28	00
-3-1-	00	00	06	08	17	06	22	14	06	03	64	33	03	33	72	67	14	33	56	31	47	25	67	69	58	44	100	64	17	53	83	31	03
-3-3-	03	00	00	00	00	00	03	44	03	03	14	39	03	03	03	81	11	03	22	36	11	17	11	78	44	67	83	100	22	64	86	58	00
-1-1-	00	00	03	19	11	03	08	39	17	17	31	06	06	19	11	44	31	17	50	42	53	56	33	47	89	92	36	19	97	78	67	06	00
-1-3-	00	00	00	03	00	00	08	25	00	17	11	06	03	42	03	39	17	67	28	58	08	58	19	42	36	42	36	61	92	97	86	67	00
-3-1-	00	06	00	03	00	03	06	19	00	08	53	25	00	06	36	56	14	08	42	42	28	28	42	86	28	31	83	86	78	61	100	75	00
-3-3-	00	03	00	06	03	03	00	03	00	00	00	19	00	00	00	00	00	03	06	64	00	00	03	58	08	42	31	83	08	69	94	100	00
	S ₁	S ₂	S ₃	S ₄	U ₁	U ₂	U ₃	U ₄	R ₁	R ₂	R ₃	R ₄	D ₁	D ₂	D ₃	D ₄	W ₁	W ₂	W ₃	W ₄	K ₁	K ₂	K ₃	K ₄	C ₁	C ₂	C ₃	C ₄	O ₁	O ₂	O ₃	O ₄	

Table 16 The percentage of "same" judgments obtained during the middle third of each experimental session for the 32 Morse-like rhythmic patterns.

	s_1	s_2	s_3	s_4	u_1	u_2	u_3	u_4	R_1	R_2	R_3	R_4	D_1	D_2	D_3	D_4	W_1	W_2	W_3	W_4	K_1	K_2	K_3	K_4	G_1	G_2	G_3	G_4	O_1	O_2	O_3	O_4	
$-1-1-s_1$	97	69	22	06	92	19	08	00	83	31	03	00	69	11	03	00	17	00	00	00	44	00	00	00	00	03	03	03	03	03	03	03	03
$-1-1-s_2$	50	92	56	17	44	86	33	17	69	58	25	00	47	47	39	17	28	36	03	03	31	22	19	00	14	11	11	06	11	00	03	03	03
$-3-1-s_3$	42	75	94	42	33	42	94	14	47	14	56	06	42	39	61	08	19	06	25	03	17	03	22	03	19	06	00	03	00	00	08	00	00
$-3-3-s_4$	03	28	47	97	00	19	42	69	00	03	58	53	00	22	22	53	08	06	14	03	08	11	14	42	03	17	17	44	11	03	14	03	03
$-1-1-u_1$	56	58	36	03	100	61	19	00	92	36	06	06	78	17	08	03	92	25	06	06	81	08	11	00	08	06	03	00	19	00	00	00	00
$-1-3-u_2$	06	75	28	06	28	97	33	28	39	42	17	03	11	58	14	08	61	69	28	08	64	42	25	11	17	14	17	11	06	06	11	08	00
$-3-1-u_3$	00	33	47	25	06	42	97	39	19	06	50	14	11	14	39	17	33	36	56	14	42	25	61	08	31	00	22	06	19	00	17	03	03
$-3-3-u_4$	00	06	00	47	00	11	50	94	03	14	42	42	03	11	06	28	08	08	61	64	14	28	28	78	31	33	14	14	17	28	33	17	00
$-1-1-R_1$	19	33	25	08	67	53	17	08	92	64	31	03	42	42	22	14	78	14	17	00	58	14	14	06	53	14	11	03	17	00	06	11	00
$-1-3-R_2$	03	36	14	11	08	33	19	11	33	97	19	17	06	47	19	33	61	78	31	08	25	28	22	06	61	50	14	00	14	08	11	03	03
$-3-1-R_3$	08	08	31	14	08	08	50	31	42	19	92	61	14	22	36	25	17	19	94	19	17	31	50	33	33	14	67	19	11	03	39	03	03
$-3-3-R_4$	03	03	06	28	00	06	19	25	08	39	61	94	06	22	11	47	14	25	39	56	00	11	06	39	22	25	47	53	08	25	58	03	03
$-1-1-D_1$	56	42	17	06	72	19	17	00	61	47	14	00	97	42	50	08	36	03	00	06	97	28	25	06	67	17	03	08	03	00	06	03	03
$-1-3-D_2$	03	47	11	17	06	31	11	17	31	25	31	11	42	94	42	64	28	31	14	06	44	92	42	33	72	67	36	06	33	28	14	00	00
$-3-1-D_3$	03	08	22	06	00	36	28	06	06	14	42	03	33	50	94	47	14	06	06	00	39	11	97	19	25	19	47	06	08	06	25	00	00
$-3-3-D_4$	00	03	03	22	03	06	25	42	00	11	14	47	03	28	64	94	14	03	19	31	06	06	53	72	11	19	72	53	19	17	64	03	03
$-1-1-W_1$	03	31	14	06	31	58	39	06	61	72	17	17	36	44	28	14	89	75	39	08	78	53	17	08	53	42	19	08	78	31	06	00	00
$-1-3-W_2$	06	06	00	03	06	53	22	28	22	72	25	11	14	36	11	25	69	92	42	19	47	64	17	19	28	67	22	03	50	53	28	06	06
$-3-1-W_3$	03	00	17	14	00	08	47	50	19	14	86	28	00	06	14	47	19	39	94	58	06	25	28	36	39	61	56	28	53	31	72	39	03
$-3-3-W_4$	00	03	03	17	06	00	00	42	03	08	14	56	03	00	08	61	06	36	64	89	00	06	14	58	14	39	31	69	28	56	64	69	03
$-1-1-K_1$	14	14	08	03	36	44	25	03	25	44	31	06	67	50	56	11	72	14	17	06	94	72	64	03	53	36	06	00	58	14	11	03	03
$-1-3-K_2$	00	17	00	00	03	08	11	03	03	25	17	06	17	72	25	14	11	47	17	14	61	97	47	33	31	33	31	22	33	69	28	08	08
$-3-1-K_3$	03	00	11	03	03	19	53	03	06	08	17	06	14	17	86	36	03	11	19	00	44	11	97	58	14	08	86	06	17	14	75	03	03
$-3-3-K_4$	00	03	06	08	03	06	06	39	00	03	06	11	06	11	25	83	03	06	14	14	11	17	75	97	06	28	58	44	25	28	78	67	03
$-1-1-G_1$	00	03	00	06	06	17	19	17	50	50	42	08	50	72	25	28	75	14	39	08	75	50	17	06	92	81	11	14	75	22	25	08	08
$-1-3-G_2$	03	17	00	08	08	08	03	11	11	42	17	25	08	56	00	47	28	47	28	17	31	31	11	33	64	94	61	58	64	72	47	06	06
$-3-1-G_3$	03	03	03	03	03	08	08	03	03	03	56	14	03	17	69	56	03	00	67	25	19	08	69	42	39	39	94	44	17	44	92	08	08
$-3-3-G_4$	00	03	00	06	03	03	03	11	03	11	11	28	00	00	08	61	03	11	19	36	06	06	11	56	19	53	69	97	17	53	75	50	03
$-1-1-O_1$	03	00	08	11	08	03	14	28	08	06	25	11	00	06	08	25	22	19	28	22	44	39	31	22	69	75	25	11	97	86	44	11	00
$-1-3-O_2$	00	00	03	00	03	06	06	06	06	00	03	11	03	08	03	14	22	39	11	28	06	61	14	42	36	69	22	50	72	94	42	50	02
$-3-1-O_3$	00	00	00	03	00	03	11	14	03	08	31	14	00	08	19	33	03	17	22	19	11	22	53	64	14	08	67	44	53	53	97	38	03
$-3-3-O_4$	00	03	03	08	03	00	00	14	00	00	00	14	03	06	00	06	00	06	06	36	00	03	06	36	08	33	25	58	06	64	92	97	04
s_1	s_2	s_3	s_4	u_1	u_2	u_3	u_4	R_1	R_2	R_3	R_4	D_1	D_2	D_3	D_4	W_1	W_2	W_3	W_4	K_1	K_2	K_3	K_4	G_1	G_2	G_3	G_4	O_1	O_2	O_3	O_4		

Table 17 The percentage of "same" judgments obtained during the final third of each experimental session for the 32 Morse-like rhythmic patterns.

	S ₁	S ₂	S ₃	S ₄	U ₁	U ₂	U ₃	U ₄	R ₁	R ₂	R ₃	R ₄	D ₁	D ₂	D ₃	D ₄	W ₁	W ₂	W ₃	W ₄	K ₁	K ₂	K ₃	K ₄	G ₁	G ₂	G ₃	G ₄	O ₁	O ₂	O ₃	O ₄	
.1-1-1-1	94	83	50	00	97	14	11	03	78	22	03	00	56	06	00	00	17	00	00	00	17	06	00	00	08	00	00	06	00	03	03	03	S ₁
.1-1-3-1	42	97	67	17	50	81	33	00	53	61	19	06	44	47	19	11	25	22	11	00	44	14	11	00	14	06	00	00	00	00	00	00	S ₂
.3-1-1-1	39	56	92	50	25	53	89	17	50	33	56	14	36	31	50	31	22	03	17	00	25	00	44	03	03	06	03	00	11	03	06	00	S ₃
.3-3-1-1	03	31	58	97	03	11	61	81	03	17	42	50	06	19	17	47	00	03	25	14	06	06	08	25	00	19	19	22	17	03	08	03	S ₄
.1-1-1-U ₁	58	42	44	00	97	75	39	00	78	39	03	03	64	25	08	03	83	08	06	08	58	08	03	00	19	06	06	00	11	00	00	03	U ₁
.1-1-3-U ₂	03	47	17	31	44	94	31	39	42	33	31	03	06	44	17	08	56	86	14	00	44	42	31	22	25	06	03	03	17	06	06	00	U ₂
.3-1-1-U ₃	08	22	64	08	08	28	92	28	19	17	42	14	25	17	58	08	39	17	61	17	50	14	56	25	31	06	28	00	31	08	06	06	U ₃
.3-3-1-U ₄	00	03	06	22	03	14	50	94	06	25	50	61	08	08	17	33	00	22	47	75	17	31	17	53	17	14	22	11	14	22	33	14	U ₄
.1-1-1-R ₁	19	36	22	06	67	50	17	03	97	57	33	11	50	42	08	11	69	28	06	03	50	06	06	03	47	08	11	03	31	00	00	03	R ₁
.1-1-3-R ₂	08	11	17	00	11	28	25	06	44	94	50	28	19	47	25	36	61	83	22	05	03	39	03	00	25	50	19	11	08	25	00	00	R ₂
.3-1-1-R ₃	00	03	06	31	03	25	42	33	44	44	94	64	08	17	50	14	17	25	78	28	19	19	44	28	36	14	47	03	17	06	53	03	R ₃
.3-3-1-R ₄	00	03	03	14	00	08	19	22	14	53	56	92	00	19	11	28	03	14	11	69	06	08	00	17	14	42	36	78	17	33	42	06	R ₄
.1-1-1-D ₁	42	33	17	03	69	66	06	03	64	33	06	00	94	14	42	03	28	11	00	00	92	19	19	00	64	25	06	00	17	03	08	00	D ₁
.1-1-3-D ₂	00	53	11	17	11	36	22	14	19	42	19	17	22	92	33	39	17	17	03	03	47	78	25	22	61	64	33	11	36	31	14	00	D ₂
.3-1-1-D ₃	00	06	31	03	03	39	19	03	06	17	28	00	17	42	89	61	06	03	19	00	36	19	100	11	25	19	61	03	03	03	22	03	D ₃
.3-3-1-D ₄	03	00	06	08	03	03	19	39	03	06	06	11	06	33	72	100	03	06	17	17	06	22	36	75	17	22	81	72	28	17	61	00	D ₄
.1-1-1-W ₁	06	14	22	08	44	53	22	28	58	75	14	14	47	25	11	08	94	64	14	06	56	36	17	11	64	50	03	06	75	19	06	00	W ₁
.1-1-3-W ₂	00	03	08	06	19	53	11	17	19	58	11	03	06	28	17	03	69	94	39	08	39	56	25	17	17	31	19	03	61	33	22	19	W ₂
.3-1-1-W ₃	00	08	06	06	03	08	44	50	08	22	61	33	00	03	22	31	22	22	92	61	08	14	28	36	19	47	42	25	36	31	78	28	W ₃
.3-3-1-W ₄	00	03	06	36	00	06	06	39	00	19	22	50	03	14	11	31	03	44	58	94	08	14	11	50	22	28	28	53	31	39	58	78	W ₄
.1-1-1-K ₁	00	19	03	03	33	22	19	06	33	25	25	06	64	44	39	14	58	36	17	06	94	69	31	08	69	42	06	00	69	11	08	00	K ₁
.1-1-3-K ₂	03	08	03	00	03	19	14	03	06	17	08	03	03	53	31	36	31	36	06	14	33	97	17	28	42	39	42	11	17	67	39	08	K ₂
.3-1-1-K ₃	00	00	17	03	08	19	42	11	03	11	06	11	06	14	72	44	17	11	08	03	61	33	97	69	14	03	69	17	28	06	72	00	K ₃
.3-3-1-K ₄	00	00	03	06	00	08	06	42	00	00	08	14	00	08	36	67	00	11	06	14	06	25	64	97	03	22	53	31	14	28	83	39	K ₄
.1-1-1-G ₁	00	17	03	06	06	08	03	14	42	53	50	22	47	78	42	36	69	19	36	06	83	50	36	31	89	92	22	11	81	36	25	03	G ₁
.1-1-3-G ₂	00	03	03	03	03	03	08	08	03	47	33	33	06	56	08	42	25	39	28	17	14	44	28	31	64	92	44	42	67	72	44	22	G ₂
.3-1-1-G ₃	00	00	06	00	03	00	03	19	00	06	50	19	08	31	61	58	06	06	53	08	19	14	36	25	31	28	94	53	08	31	42	11	G ₃
.3-3-1-G ₄	00	03	00	08	00	06	03	11	00	08	08	39	00	00	11	44	03	11	39	28	00	11	03	31	14	58	69	94	14	25	78	44	G ₄
.1-1-1-O ₁	03	00	03	03	11	06	11	25	33	14	19	08	06	22	08	31	42	25	36	28	56	56	28	14	69	81	08	11	97	83	61	11	O ₁
.1-1-3-O ₂	00	00	03	03	00	06	08	14	06	06	08	17	03	22	03	39	25	58	17	56	11	56	08	39	33	56	17	42	75	92	53	50	O ₂
.3-1-1-O ₃	00	00	03	00	00	03	14	11	06	11	25	03	00	08	28	58	08	25	22	19	14	17	56	69	17	22	72	75	58	42	97	53	O ₃
.3-3-1-O ₄	00	00	06	03	00	00	03	11	00	00	00	17	03	00	03	08	00	08	06	25	03	11	00	64	06	31	22	83	14	58	81	97	O ₄
	S ₁	S ₂	S ₃	S ₄	U ₁	U ₂	U ₃	U ₄	R ₁	R ₂	R ₃	R ₄	D ₁	D ₂	D ₃	D ₄	W ₁	W ₂	W ₃	W ₄	K ₁	K ₂	K ₃	K ₄	G ₁	G ₂	G ₃	G ₄	O ₁	O ₂	O ₃	O ₄	

are based solely on responses by Ss during the indicated block of experimental presentations.

The percentage of confusions decreased from 30.8% in the first third to 24.1% in the middle third to 23.1% in the final third of the experiment. However, it is important to note that the percentage of "same" responses to pairs in which the two signals were the same also decreased somewhat--from 97% to 95% to 94.4%. Thus, it appears that Ss learn that the signals of most pairs (91.2%) are different and change their criterion appropriately.

I would like to point out, however, that the decrease in the confusion rate is greater for certain kinds of pairs than for others. For convenience I shall refer to pairs (x,y) in which $x < y$ or $y < x$ as dominance pairs and to pairs (x,y) , $x \neq y$, in which neither $x < y$ nor $y < x$ as nondominance pairs. The following is a description of the processes involved in confusing signals of both kinds of pairs.

If (x,y) is a dominance pair, then by definition, each component of the shorter signal is less than or equal to the corresponding component of the longer signal, and at least one component of the shorter signal is less than the corresponding component of the longer signal. Accordingly, if a S confuses the signals of a dominance pair, he has failed to observe that certain components of one signal are shorter than the corresponding components of the other signal.

If (x,y) is a nondominance pair, then at least one component of each signal is greater than the corresponding component of the other. For example, $(.1-1-, -1.1-)$, $(.1.3-, -1-3.)$, and $(-1.3., .3-1-)$ are nondominance pairs. Observe that the signals of the first of these pairs are permutations of each other, and that the signals of the second and third pairs can be made more alike by permuting components.

In general, one of the most important ways in which signals of non-dominance pairs differ is in the particular order in which the components occur. It appears, then, that a S who confuses the signals of a non-dominance pair has failed to observe sequential differences between the signals.

Of the 992 pairs of nonidentical signals, 570 are nondominance pairs and 422 are dominance pairs. The mean percentage of "same" responses to nondominance pairs decreases from 32.3% for the first third of the experimental trials to 22.9% for the middle third of the trials to 21.7% for the final third of the trials. The corresponding means for the dominance pairs are 28.9%, 25.8%, and 24.9%. Thus, while the confusion rate for nondominance pairs in the final third of the experiment is only 67% of its rate in the first third, the confusion rate for dominance pairs in the final third of the experiment is 86% of its rate in the first third. These results suggest that in the course of the experimental session Ss learn to detect sequential differences relatively better than differences in component duration. Shepard (1963, p. 47) describes a somewhat similar effect associated with learning to identify the signals of the IMC.

Despite the effect described above, the basic patterns of confusion do not appear to change much during the course of the experiment. Although it is difficult to see this in Tables 15 - 17, we can observe this consistency in the submatrices shown in Tables 18 and 19. Notice the simplex structure in all three matrices of Table 18 and the double simplex structure in the matrices of Table 19. The presence of these predicted patterns indicates that the hypotheses are supported by the data from each third of the experiment.

TABLE 18

An Approximate Simplex in the Matrices
for Each Third of the Experimental Session

A. First Third of Experimental Session

	S ₁	S ₂	R ₂	G ₂	G ₄	O ₄
• 1 • 1 • S ₁	100	83	33	03	00	00
• 1 • 3 • S ₂	42	94	56	06	03	03
• 1 • 3 • R ₂	03	47	97	58	06	00
- 1 • 3 • G ₂	00	06	25	94	72	28
- 3 • 3 • G ₄	03	00	03	67	100	58
- 3 • 3 • O ₄	00	03	00	42	83	100

B. Middle Third of Experimental Session

	S ₁	S ₂	R ₂	G ₂	G ₄	O ₄
• 1 • 1 • S ₁	97	69	31	03	03	03
• 1 • 3 • S ₂	50	92	55	11	06	03
• 1 • 3 • R ₂	03	36	97	50	00	03
- 1 • 3 • G ₂	03	17	42	94	58	06
- 3 • 3 • G ₄	00	03	11	53	97	50
- 3 • 3 • O ₄	00	03	00	33	58	97

C. Final Third of Experimental Session

	S ₁	S ₂	R ₂	G ₂	G ₄	O ₄
• 1 • 1 • S ₁	94	83	22	00	06	03
• 1 • 3 • S ₂	42	97	61	06	00	00
• 1 • 3 • R ₂	08	11	94	50	11	00
- 1 • 3 • G ₂	00	03	44	92	42	22
- 3 • 3 • G ₄	00	03	08	58	94	44
- 3 • 3 • O ₄	00	00	00	31	83	97

TABLE 19

An Approximate Double Simplex in the Matrices
for Each Third of the Experimental Session

A. First Third of Experimental Session

	D ₁	K ₁	K ₂	K ₄	R ₁	W ₁	W ₂	W ₄
-1-1- D ₁	97	94	31	00	83	36	19	03
-1-1- K ₁	78	97	81	06	42	75	28	08
-1-3- K ₁	03	64	100	50	06	25	61	17
-3-3- K ₂	03	06	39	100	00	03	17	19
					1	2		
					3	4		
•1-1- R ₁	58	78	14	06	94	69	47	00
•1-1- W ₁	42	94	58	08	72	97	69	06
•1-3- W ₁	08	31	72	19	31	47	94	14
•3-3- W ₂	00	08	08	42	06	08	33	94

B. Middle Third of Experimental Session

	D ₁	K ₁	K ₂	K ₄	R ₁	W ₁	W ₂	W ₄
-1-1- D ₁	97	97	28	06	61	36	03	06
-1-1- K ₁	67	94	72	03	25	72	14	06
-1-3- K ₁	17	61	97	33	03	11	47	14
-3-3- K ₂	06	11	17	97	00	03	06	14
					1	2		
					3	4		
•1-1- R ₁	42	42	14	06	92	78	14	00
•1-1- W ₁	36	78	53	06	61	89	75	08
•1-3- W ₁	14	47	64	19	22	69	92	19
•3-3- W ₂	03	00	06	58	03	06	36	89

C. Final Third of Experimental Session

	D ₁	K ₁	K ₂	K ₄	R ₁	W ₁	W ₂	W ₄
-1-1- D ₁	94	92	19	00	64	28	11	00
-1-1- K ₁	64	94	69	08	33	58	36	06
-1-3- K ₁	03	33	97	28	06	31	36	14
-3-3- K ₂	00	06	25	97	00	00	11	14
					1	2		
					3	4		
•1-1- R ₁	50	50	06	03	97	69	28	03
•1-1- W ₁	47	56	36	11	58	94	64	06
•1-3- W ₁	06	39	56	17	19	69	94	08
•3-3- W ₂	03	08	14	50	00	03	44	94

If the matrices in Tables 15, 16, and 17 are similar in structure, we should also expect high correlations between corresponding rows of the three matrices as well as between corresponding columns of the three matrices. The average correlations between corresponding rows and between corresponding columns of Table 15 and 16, Tables 15 and 17, and Tables 16 and 17 are given below:

Tables 15 and 16 (first and middle thirds)

Rows -- $\bar{r} = .893$

Columns -- $\bar{r} = .891$

Tables 15 and 17 (first and final thirds)

Rows -- $\bar{r} = .884$

Columns -- $\bar{r} = .881$

Tables 16 and 17 (middle and final thirds)

Rows -- $\bar{r} = .923$

Columns -- $\bar{r} = .922$

These high correlations are evidence of considerable stability of the structure of the confusion matrix over the three stages of the experiment. Thus, despite the fact that the criterion changes, the patterns of confusion remains quite constant.

D. Multidimensional Configurations for the Morse-like Rhythmic Patterns

The configuration shown in Fig. 6 is the 2-dimensional row solution (coefficient of alienation = .17) obtained by means of G-L(SSA-II). The input to the computer was the total confusion data for the Morse-like rhythmic patterns. Observe that the vertical dimension orders the signals according to temporal length. Thus, the shortest signal, (.1.1.), is in the lowest band, whereas the longest signal is in the highest band. Furthermore, the signals within any band have a greater temporal length than those in the band below it and a shorter temporal length than those

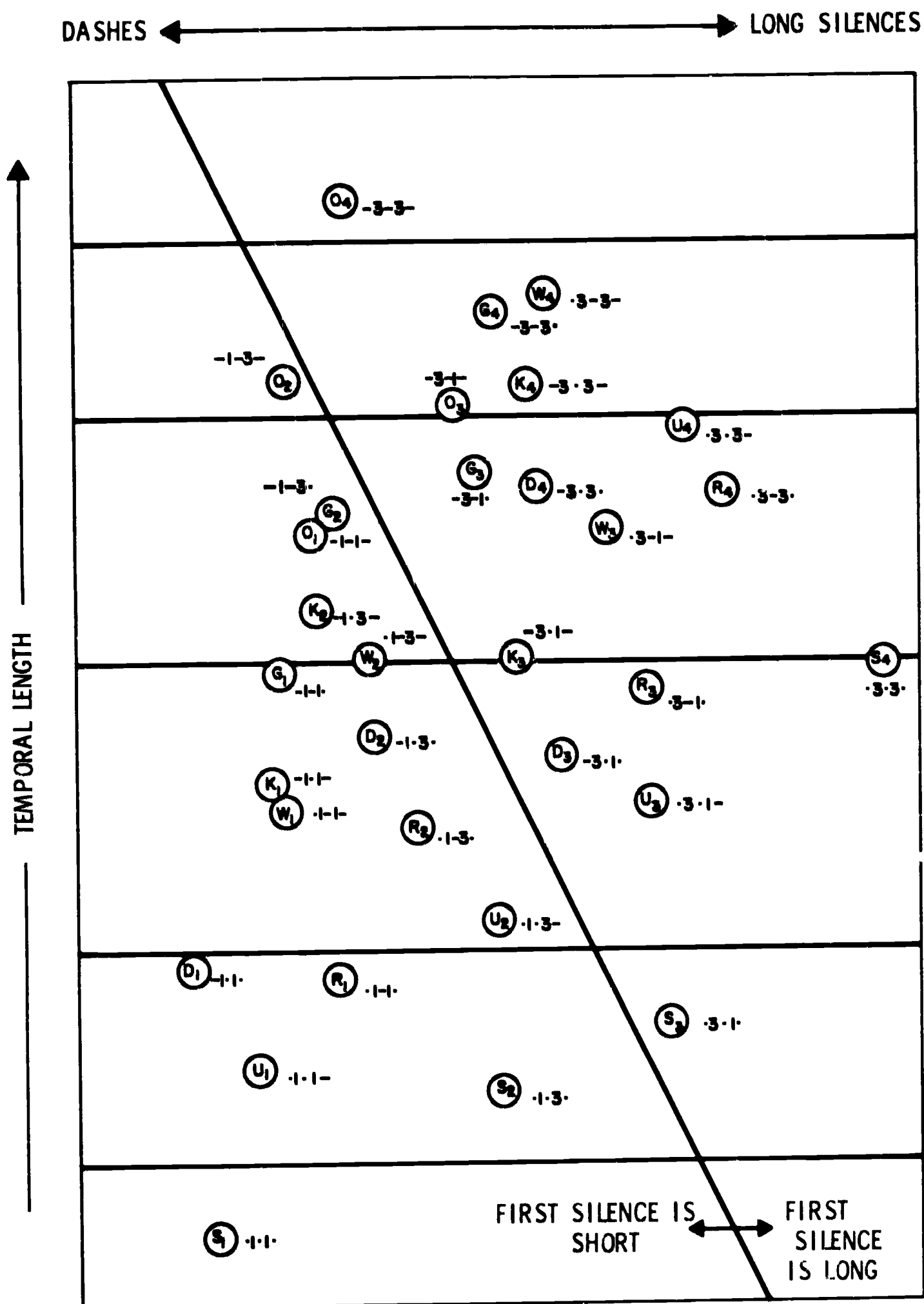


Fig. 6. A two-dimensional configuration for the 32 Morse-like rhythmic patterns - I. [Guttman-Lingoes (Smallest Space Analysis - II) row solution]

in the band above it. Within each band the signals with more dashes and more short silences are generally to the left, whereas the signals with more dots and more long silences are to the right. Accordingly, the second dimension reflects the sound-to-silence ratio for signals having the same temporal length.

It has previously been pointed out that signals differing on only the second component, i.e., the first silence, are easier to distinguish than signals which differ on any other single component. Observe that a diagonal line divides the signals on the basis of their second component. All signals whose second component is a short silence are to the left of the diagonal, whereas all signals whose second component is a long silence are to the right of the diagonal.

It has also been previously stated that signals differing on the final component alone are more likely to be confused than signals differing on any other single component. Observe that signals differing on only the last component are usually closer to each other in the figure than signals differing from each other on any other single component.

The configuration in Fig. 7 is the same as that in Fig. 6. I have, however, omitted the boundary lines in Fig. 7 and have grouped together all signals that differ only in an inversion of components. Signals within the same oval have the same temporal length, the same number of dashes, the same number of dots, the same number of long silences, and the same number of short silences. Note that groups of signals with the same number of dashes are ordered on the basis of number of long silences. For example, for each of the three groups of signals with the same number of dashes, the group of signals with one long silence is between the group with no long silences and the group

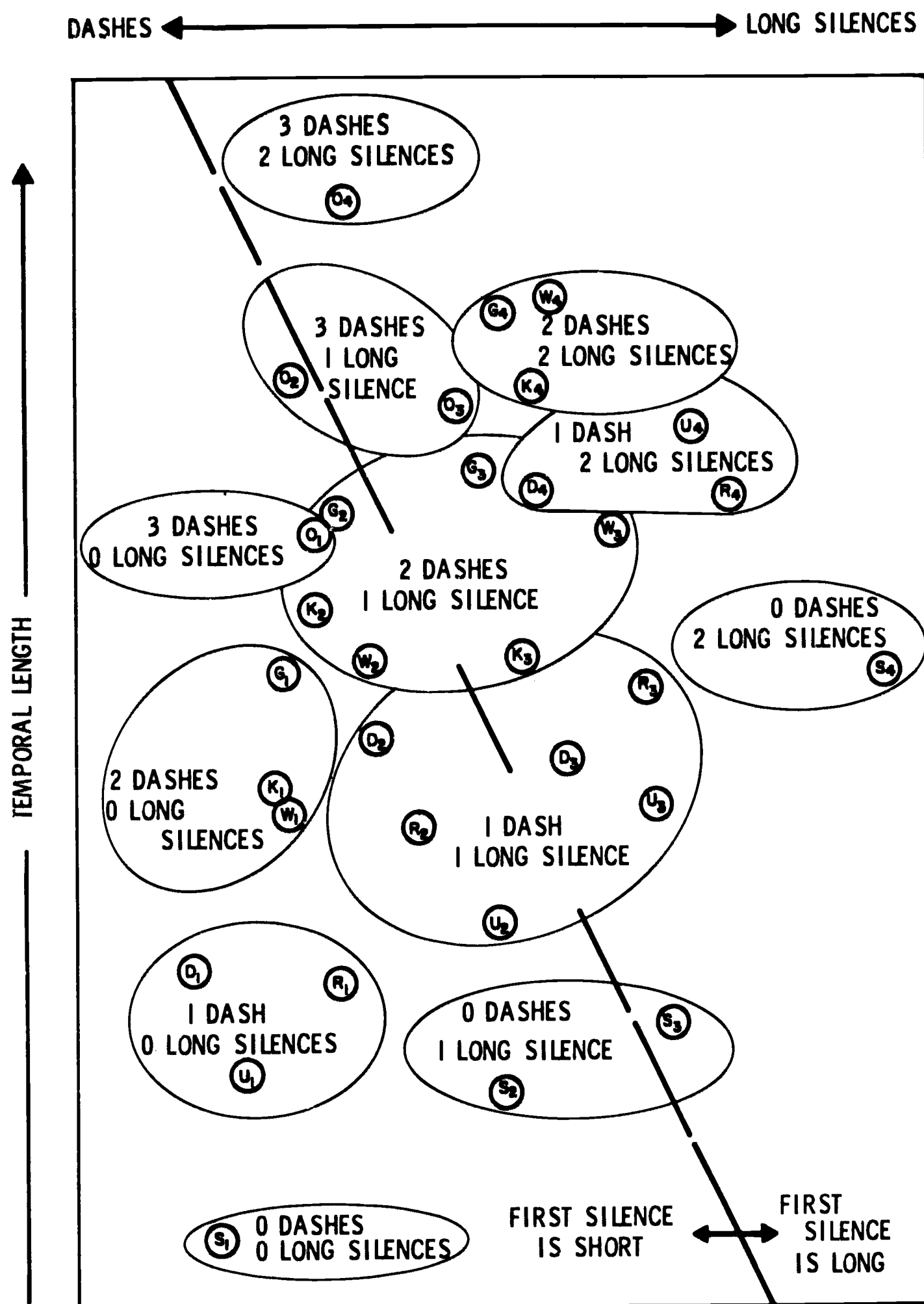


Fig. 7. A two-dimensional configuration for the 32 Morse-like rhythmic patterns - II. [Guttman-Lingoes (Smallest Space Analysis - II) row solution]

with two long silences. Similarly, the groups of signals that have the same number of long silences are ordered in Fig. 7 according to the number of dashes they have.

The two-dimensional configuration shown in Fig. 8 is the solution (stress = .23) obtained for the same data by means of the Kruskal program with the nonsymmetric option. This configuration, as well as the two-dimensional solution obtained by means of the Kruskal procedure with the symmetric option (stress = .19), the two dimensional solution obtained by means of the Guttman-Lingoes symmetric program, G-L(SSA-I), (Lingoes, 1965a), (coefficient of alienation = .20), and the two-dimensional column solution obtained by means of G-L(SSA-II), (coefficient of alienation = .20), is extremely similar to the one shown in Figs. 6 and 7, the G-L(SSA-II) row solution.

The three-dimensional solutions obtained by the several procedures mentioned above are also very similar to each other. Only the G-L(SSA-II) column solution (coefficient of alienation = .13) is shown in this paper. The first and second dimensions of this configuration are shown in Fig. 9, the first and third dimensions are shown in Fig. 10, and the second and third dimensions are shown in Fig. 11.

One can observe that Fig. 9 has the same basic structure as Figs. 6, 7, and 8. Thus, it can be seen that (1) the signals are ordered according to temporal length; (2) within each band of constant temporal length the signals with the most dashes and short silences are to the left, whereas the signals with the most dots and most long silences are to the right; and (3) a diagonal line separates the signals with a short first silence from the signals with a long first silence.

In Fig. 10 all signals to the left of the diagonal begin with a dash, whereas all signals to the right of the diagonal begin with a

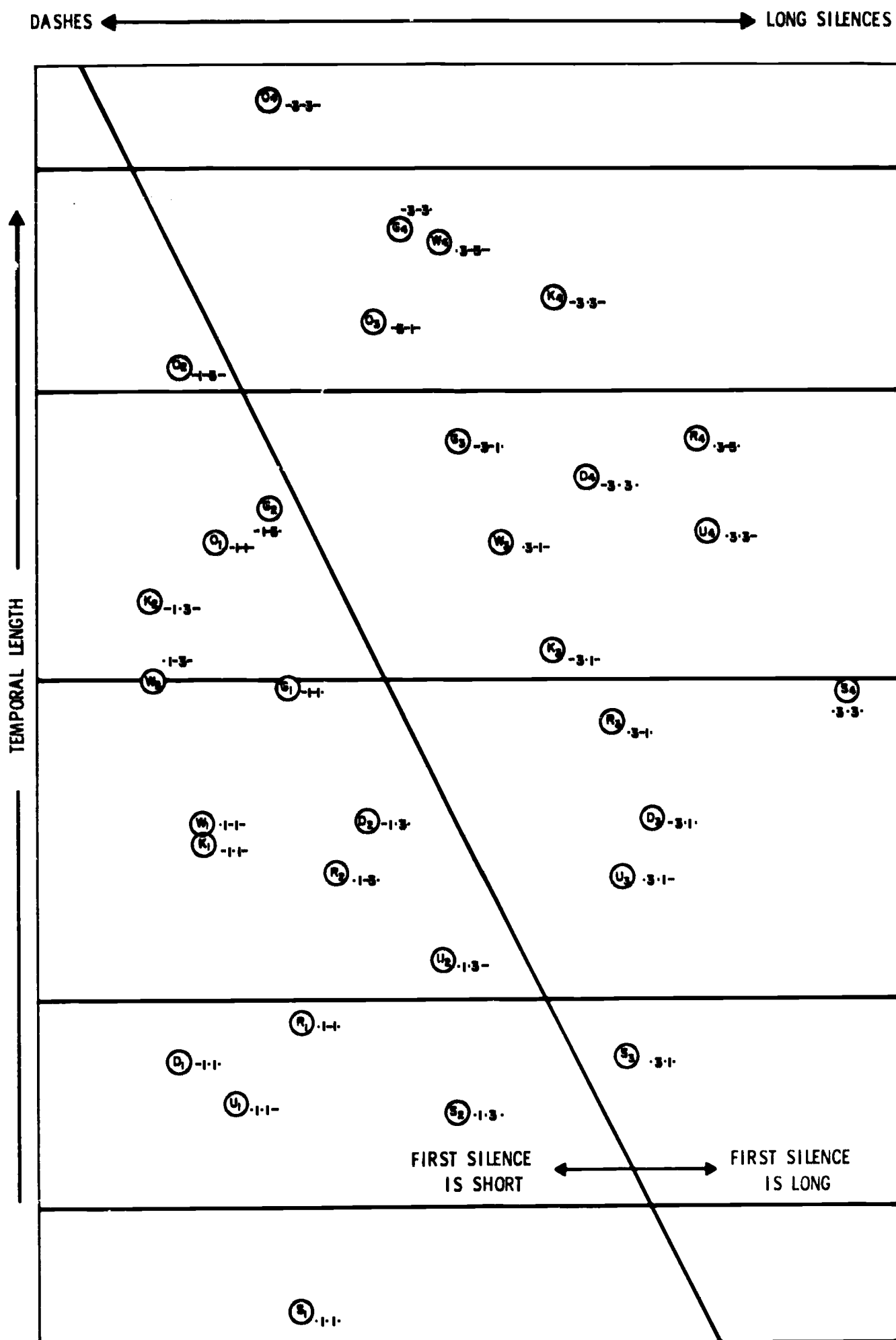


Fig. 8. A two-dimensional configuration for the 32 Morse-like rhythmic patterns - III. [Kruskal nonsymmetric solution]

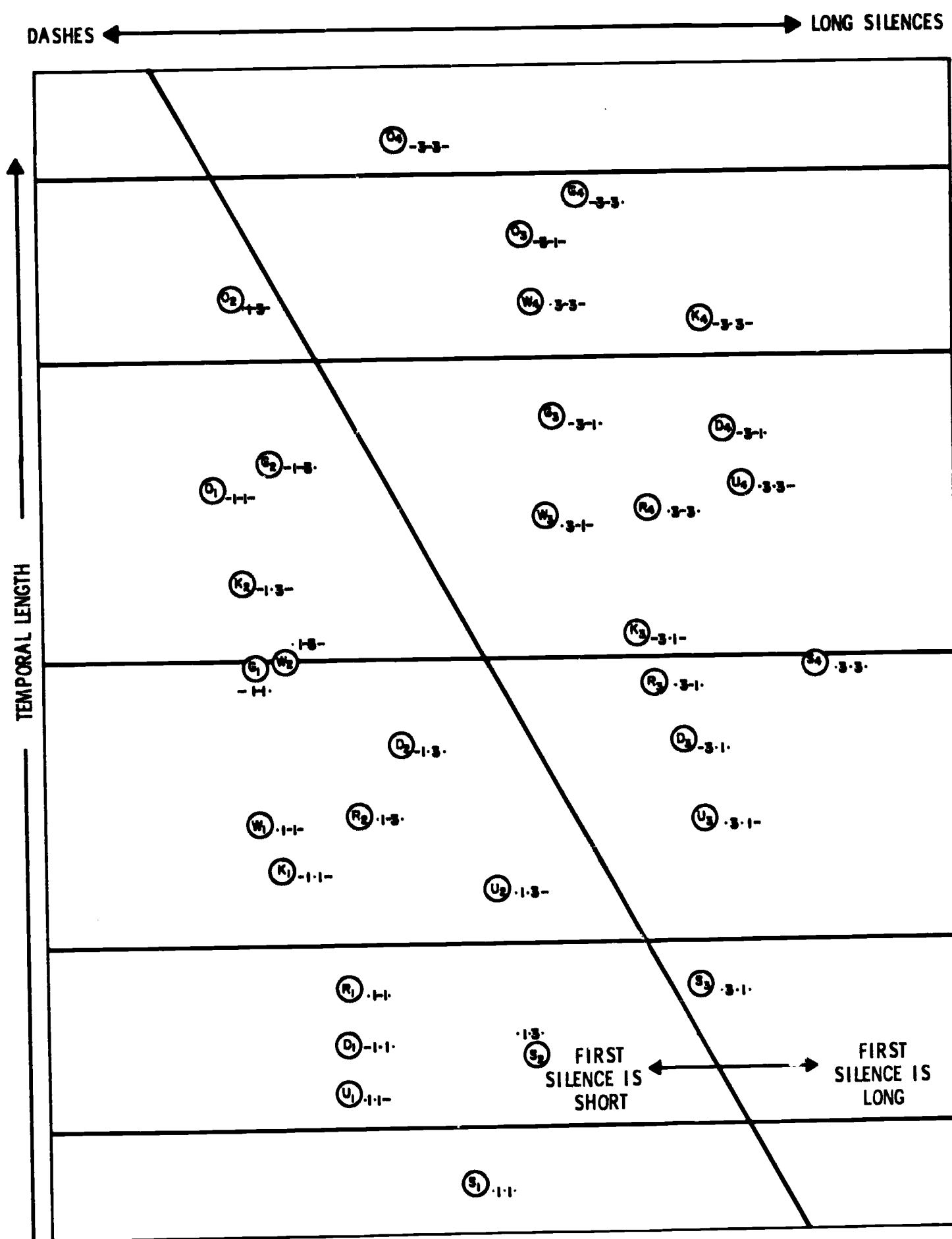


Fig. 9. First and second dimensions of a three-dimensional configuration for the 32 Morse-like rhythmic patterns. [Guttman-Lingoes (Smallest Space Analysis - II) column solution]

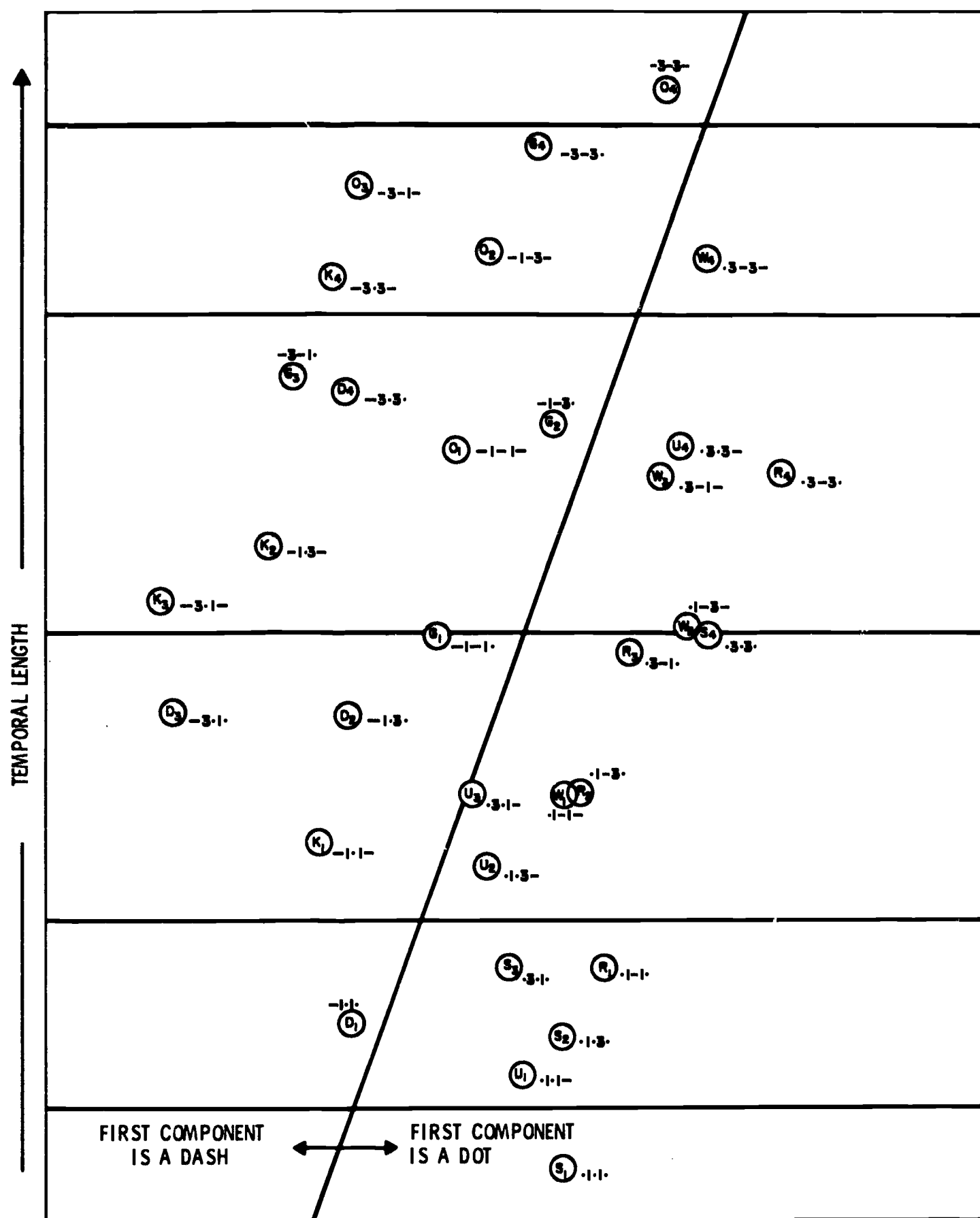


Fig. 10. First and third dimensions of a three-dimensional configuration for the 32 Morse-like rhythmic patterns. [Guttman-Lingoes (Smallest Space Analysis - II) Column Solution]

dot. Recall that differences on the first component are easier to detect than differences on any component other than the second.

In Fig. 11 the signals are divided into four groups on the basis of their first two components. The only signals out of place are O_4 and G_2 . Figure 12 also shows the second and third dimensions of the configuration, but in this figure the signals beginning with three long components (two dashes separated by a long silence) and the signals beginning with three short components (two dots separated by a short silence) are omitted. Observe that signals with the same first three components are close together in the figure. Furthermore, there is a cyclical pattern among the first three components of these signals, as each boundary separates signals which differ on one of the first three components. Thus, whereas the first dimension groups and orders signals according to their temporal length, the second and third dimensions group signals according to the particular combination of components in the signals and the precise order in which the components occur in the signals.

Separate two- and three-dimensional analyses were also obtained for the data from each third of the experiment. The two-dimensional Kruskal nonsymmetric solutions obtained for the data from the first third of the experiment (stress = .264) and for the data from the final third of the experiment (stress = .280) are shown in Figs. 13 and 14, respectively. Since these configurations (as well as the 3-dimensional configurations) are so similar to those obtained for the total data, they will not be discussed.

In the next chapter the models for both sets of signals will be integrated into a single more general structural theory.

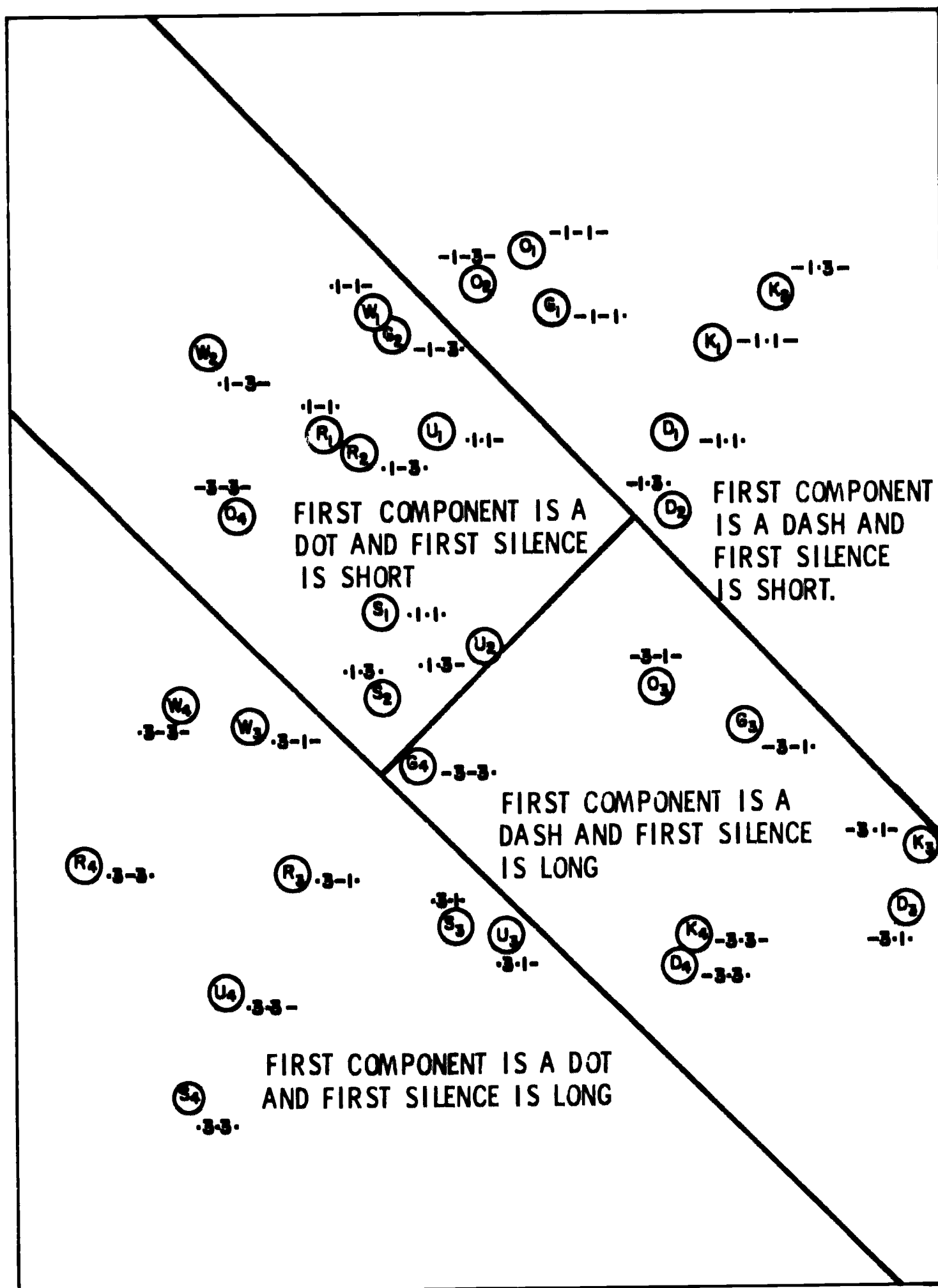


Fig. 11. Second and third dimensions of a three-dimensional configuration for the 32 Morse-like rhythmic patterns - I. [Guttman-Lingoes (Smallest Space Analysis - II) column solution]

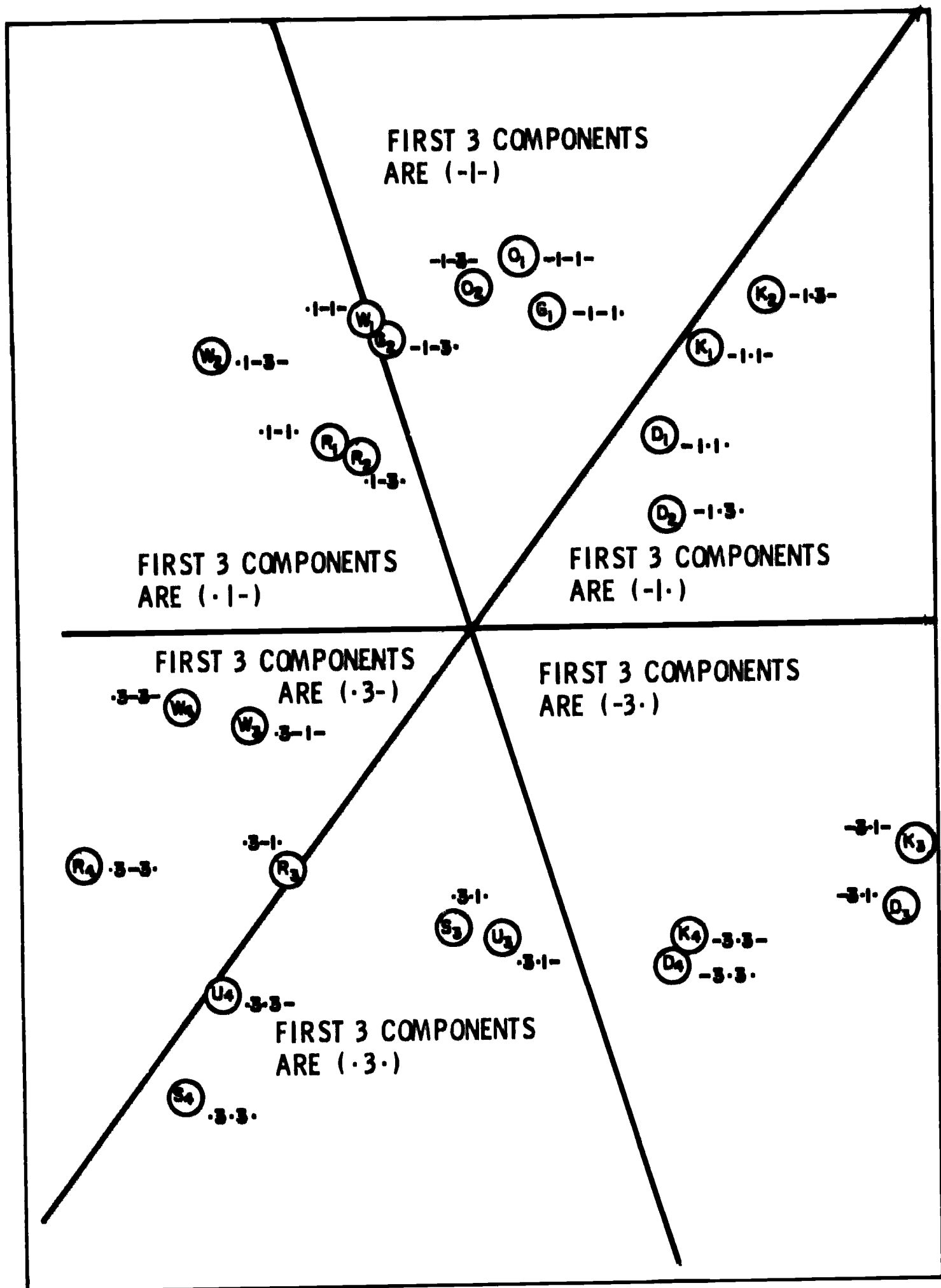


Fig. 12. Second and third dimensions of a three-dimensional configuration for the 32 Morse-like rhythmic patterns - II (signals beginning with .1. or -3- are omitted). [Guttman-Lingoes (Smallest Space Analysis - II) column solution]

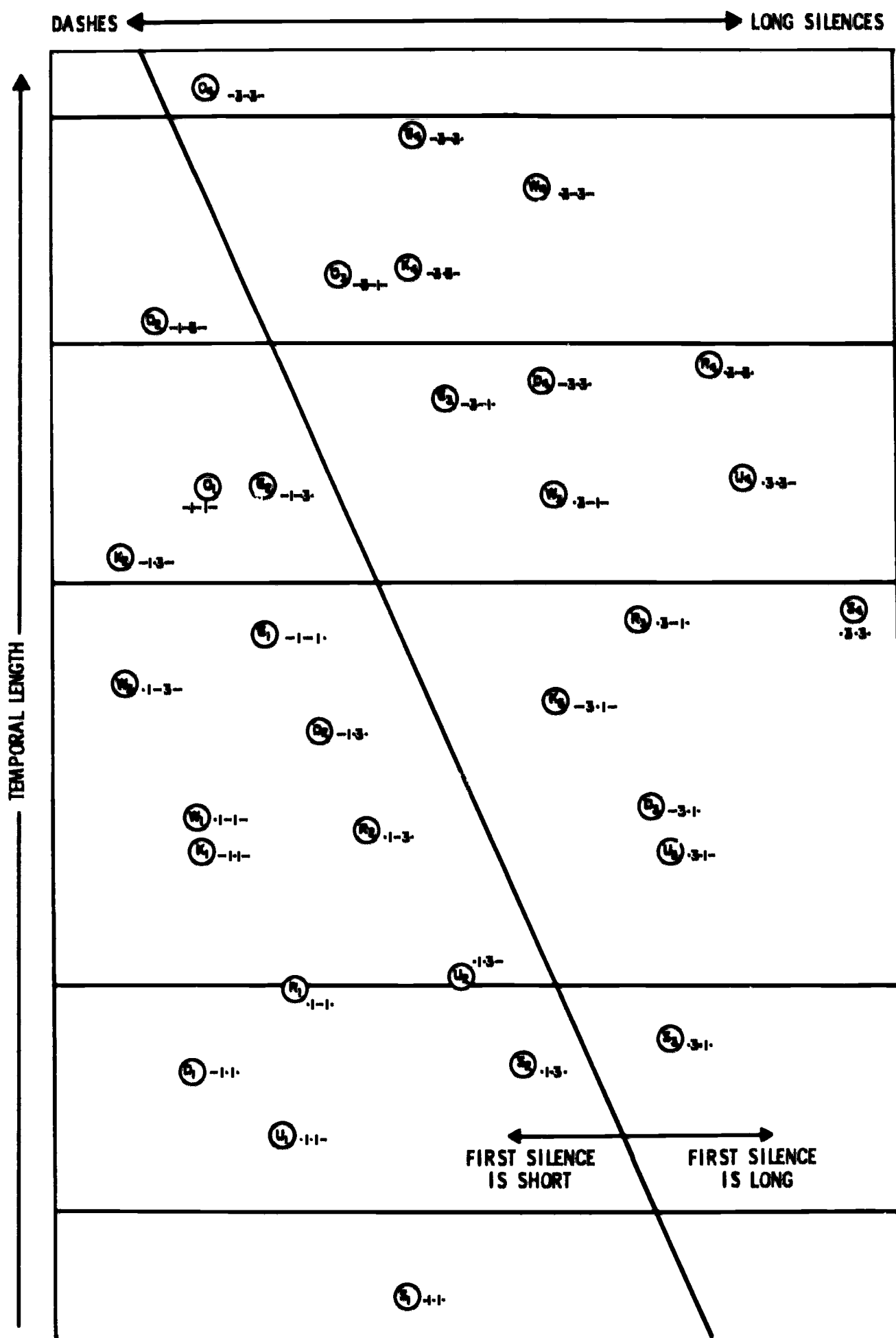


Fig. 13. A two dimensional configuration for the 32 Morse-like rhythmic patterns based on data from the first third of the experiment. [Kruskal nonsymmetric solution]

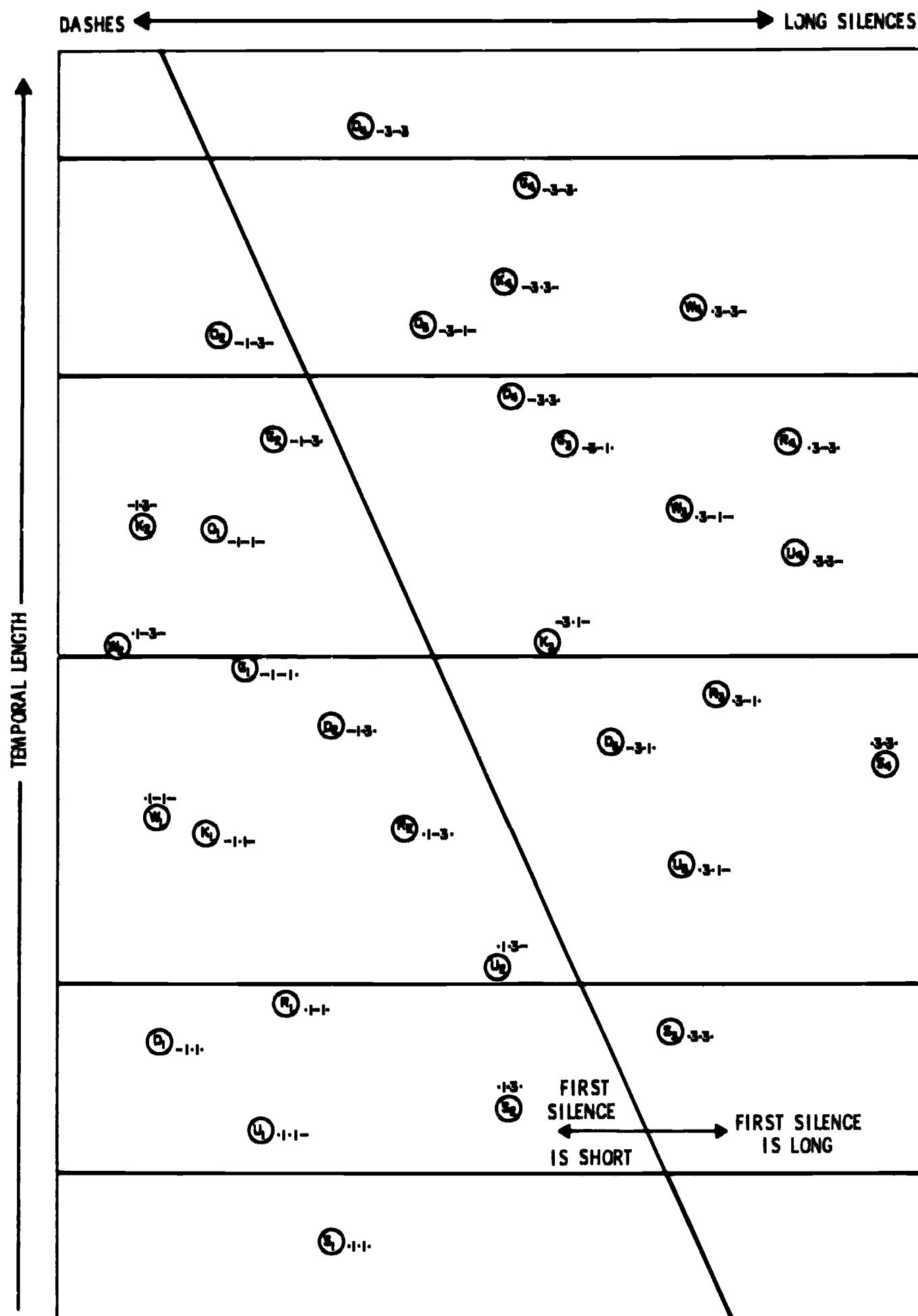


Fig. 14. A two-dimensional configuration for the 32 Morse-like rhythmic patterns based on data from the final third of the experiment. [Kruskal nonsymmetric solution]

CHAPTER 4. AN INTEGRATION OF THE FIRST AND SECOND MODELS

I. The Universe of Content

The signals first studied here varied in number of components, but all silent intervals in these signals were of the same duration. In contrast, all the signals studied in the second model had the same number of components, but the silent intervals varied in duration. Let us now consider a more general set of signals, μ_3 , which includes μ_1 and μ_2 as subsets. In Cartesian product notation $\mu_3 = \Delta \cup \Delta \times \Sigma \times \Delta \cup \Delta \times \Sigma \times \Delta \times \Sigma \times \Delta \cup \Delta \times \Sigma \times \Delta \times \Sigma \times \Delta \times \Sigma \times \Delta \cup \Delta \times \Sigma \times \Delta \times \Sigma \times \Delta \times \Sigma \times \Delta \times \Sigma \times \Delta$ where Δ is a set whose elements are a 1-unit tone (a dot) and a 3-unit tone (a dash), and Σ is a set whose elements are a 1-unit silence (a short silence) and a 3-unit silence (a long silence). Since Δ and Σ have two elements each, $\Delta \times \Sigma \times \Delta$ has $2^3 = 8$ elements (four of which are in μ_1), $\Delta \times \Sigma \times \Delta \times \Sigma \times \Delta$ has $2^5 = 32$ elements (the elements of μ_2), $\Delta \times \Sigma \times \Delta \times \Sigma \times \Delta \times \Sigma \times \Delta$ has $2^7 = 128$ elements (sixteen of which are in μ_1) and $\Delta \times \Sigma \times \Delta \times \Sigma \times \Delta \times \Sigma \times \Delta \times \Sigma \times \Delta$ has $2^9 = 512$ elements (thirty-two of which are in μ_1). Since μ_3 has so many elements, 682, I shall not provide a complete list of these signals.

II. The Structural Hypotheses

The rationale for the first three hypotheses is the same as that given in the first and second models. The fourth and fifth hypotheses are motivated by the results of the experiments discussed in this thesis. In all five hypotheses $p(x,y)$ denotes the percentage of "same" responses to the ordered pair (x,y) in the kind of task carried out by Rothkopf and the present investigator.

A. The First Hypothesis

Definition 1.

For all $x, y \in \mu_3$, $x < y$ iff it is possible to transform x to y by applying to x one or more of the following operations:

- (1) changing at least one dot to a dash;
- (2) changing at least one 1-unit silence to a 3-unit silence;
- (3) adding at least one tone and an equal number of silences.

The ordering among the signals of μ_1 and the ordering among the signals of μ_2 are identical to those established for these sets in the first and second models, respectively.

Definition 2.

For all $x, y, z, \in \mu_3$, x is closer to y than to z in the ordering on μ_3 iff $x < y < z$ or $z < y < x$.

Hypothesis 1.

For all $x, y, z \in \mu_3$, if x is closer to y than to z in the ordering on μ_3 ,

then $p(x, y) > p(x, z)$;
 $p(x, y) > p(z, x)$;
 $p(y, x) > p(z, x)$; and
 $p(y, x) > p(x, z)$.

In the results section of Chapter 2 some limitations on the first hypothesis are given.

B. The Second Hypothesis

In the following definitions $d(x)$, $s(x)$, and $t(x)$ denote the number of dashes in x , the number of long silences in x , and the temporal length of x , respectively.

Definition 3.

For all $x, y, z \in \mu_3$, y is component-wise between x and z iff:

- (1) x , y , and z have the same number of components, and
- (2) for all i , the i^{th} component of y is the same as the i^{th} component of x and/or the i^{th} component of z .

Definition 4.

For all $x, y, z \in \mu_3$, y is dash-wise between x and z iff either $d(x) \leq d(y) \leq d(z)$ or $d(z) \leq d(y) \leq d(x)$.

Definition 5.

For all $x, y, z \in \mu_3$, y is silence-wise between x and z iff either $s(x) \leq s(y) \leq s(z)$ or $s(z) \leq s(y) \leq s(x)$.

Definition 6.

For all $x, y, z \in \mu_3$, y is time-wise between x and z iff either $t(x) \leq t(y) \leq t(z)$ or $t(z) \leq t(y) \leq t(x)$.

Hypothesis 2.

For all $x, y, z \in \mu_3$, $(x \neq y \neq z)$,

if (i) y is component-wise between x and z ,

(ii) y is dash-wise between x and z ,

(iii) y is silence-wise between x and z ,

(iv) y is time-wise between x and z , and in addition,

(v) neither $x < y < z$ nor $z < y < x$,

then $p(x,y) > p(x,z)$,

$p(x,y) > p(z,x)$,

$p(y,x) > p(z,x)$, and

$p(y,x) > p(x,z)$.

C. The Third, Fourth, and Fifth Hypotheses

Hypothesis 3.

For all $x, y \in \mu_3$, $(x \neq y)$, $p(x,x) > p(x,y)$ and $p(y,x)$.

Hypothesis 4.

- If (i) x , y , and z ($x, y, z \in \mu_3$) have the same number of components
(ii) x and y differ only on the final component and
(iii) x and z differ on only one component, but not on the final component,

then $p(x,y) > p(x,z)$,

$p(x,y) > p(z,x)$,

$p(y,x) > p(z,x)$, and

$p(y,x) > p(x,z)$.

Hypothesis 5.

- If (i) $x < y$, ($x, y \in \mu_3$),

(ii) $d(x) < d(y)$ and/or x has fewer components than y , and

(iii) the temporal length of x is either two or four units shorter than the temporal length of y ,

then $p(x,y) > p(y,x)$.

III. Generalizations

I would predict that the five hypotheses stated above would hold for visual as well as for auditory signals. The signals could be presented visually by letting short and long light flashes represent the dots and dashes, and by letting short and long intervals between light flashes represent the short and long silent intervals. The multidimensional configuration for signals (or a sample of signals) in both modalities would probably reveal a "temporal length" dimension, a "number of components" dimension, and a "ratio of sound to silence" dimension.

It would be interesting to explore the extent to which the confusion structure is affected by the particular language spoken by the

Ss. We might find that Ss who speak a language in which the last syllable is often stressed would be able to detect differences on the final component better than differences on other components. We might also discover that the patterns of signal confusion for Ss who speak a language with little stress (such as French or Spanish) would differ considerably from the patterns of confusions for Ss who speak English. Since the signals of μ_3 can be presented to Ss who speak any language, an investigation of such group differences in the patterns of signal confusion is quite feasible.

Summary,
Chapter V,
pages 88 - 89,
available from author on request.

APPENDIX

Instructions Given to Ss Immediately before Administration of the Stimulus Lists

May I please have your attention? This is an experiment to determine how well certain signals can be told apart. You will be hearing signals which will consist of three tones each. Each tone may be either short or long. This is a short tone (.). This is a long tone (-). Most of the signals which you will hear contain both short and long tones. For example, the following signal will be presented in the experiment (-3-3.). That signal consisted of two long tones followed by a short tone.

The signals will be presented in the following way: first, one 3-toned signal, and then a pause, and then the second 3-toned signal. For example, here is a pair of signals, each with three tones--(-1.1., -1.1.). Think to yourselves whether the second signal of 3 tones sounded exactly like the first signal of 3 tones.

In this experiment you will hear many pairs of signals, and for each pair you will be asked to decide whether the second signal in the pair is the same as the first signal in the pair. Now take a look at your answer sheets. You will notice that there are two answer spaces to the right of each item. These answer spaces are labelled "Y" and "N" for "Yes" and "No" as well as "T" and "F" for "True" and "False." In this experiment when you think that the second signal is exactly the same as the first signal, you should make a mark in the Y-column for "Yes, it is the same as the first signal." If, however, the second signal does not sound exactly like the first signal, mark the item "N" for "No, it is not the same as the first signal."

The example which you were given before will now be repeated-- (-1.1., -1.1.). I think you will agree that the second signal sounded exactly like the first signal. Therefore, you should mark item 1 "Yes" on your answer sheet. Observe that item 1 is marked "Yes" on the blackboard at the front of the room.

Here is example 2--(-3.3., .3.3-). Although each of these signals had one long and two short tones, the order of short and long tones in the second signal was not the same as the order of short and long tones in the first signal. Since the second signal did not sound exactly like the first signal, item 2 is marked "No" on the blackboard. Now mark item 2 "No" on your answer sheets.

Here is example 3--(.1.1., .3.3.). Although each of these signals consisted of three short tones, the second signal was not exactly the same as the first signal. In the first signal the tones were separated by short silent periods, whereas in the second signal the tones were separated by long silent periods. Since the second signal did not sound exactly like the first signal, you should mark item 3 "No" on your answer sheet. Observe that item 3 is marked "No" on the blackboard.

Here is the fourth and final example--(.3-1., .3-1.). Since the second signal sounded exactly like the first signal, item 4 is marked "Yes" on the blackboard. Now mark item 4 "Yes" on your answer sheets.

Are there any questions?

Now get ready for the experiment. To help you keep track of the item number you should be on, the item number will be announced after every group of 15 signal pairs. There will only be three seconds between items, so answer each item as quickly as possible.

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LIST OF TABLES

Table		Page
1	The signal universe for model 1.....	4
2	Per cent "same" judgments obtained by Rothkopf for all ordered pairs of signals from the "International Morse" Code. I. Signals grouped according to temporal length...	18
3	Per cent "same" judgments obtained by Rothkopf for all ordered pairs of signals from the Morse Code. II. Signals grouped according to number of components.....	19
4	Two approximate simplexes in Rothkopf's data matrix.....	21
5	Two approximate double simplexes in Rothkopf's data matrix.....	23
6	Percentage of "same" responses to pairs of signals which differ on a single component.....	25
7	Third dimension of the three-dimensional G-L(SSA-II) row solution.....	38
8	The signal universe for model 2.....	42
9	The nine lists used in the experiment.....	48
10	Percentage of "same" judgments obtained for all ordered pairs of the 32 Morse-like rhythmic patterns. Signals grouped according to dot-and-dash pattern.....	53
11	Percentage of "same" judgments obtained for all ordered pairs of the 32 Morse-like rhythmic patterns: Signals grouped according to silence patterns.....	54
12	Percentage of "same" judgments obtained for all ordered pairs of the 32 Morse-like rhythmic patterns. Signals grouped according to temporal length.....	55
13	Two approximate simplexes in the total data matrix.....	58
14	Two approximate double simplexes in the total data matrix.....	60
15	The percentage of "same" judgments obtained during the first third of each experimental session for the 32 Morse-like rhythmic patterns.....	64

LIST OF TABLES (continued)

Table		Page
16	The percentage of "same" judgments obtained during the middle third of each experimental session for the 32 Morse-like rhythmic patterns.....	65
17	The percentage of "same" judgments obtained during the final third of each experimental session for the 32 Morse-like rhythmic patterns.....	66
18	An approximate simplex in the matrices for each third of the experimental session.....	69
19	An approximate double simplex in the matrices for each third of the experimental session.....	70

LIST OF FIGURES

Figure	Page
1. A two-dimensional configuration of all 36 International Morse Code signals based upon the same-or-different judgments obtained by Rothkopf - I. [Kruskal symmetric solution - from Shepard, 1963, p. 39].....	30
2. A two-dimensional configuration of all International Morse Code signals based upon the same-or-different judgments obtained by Rothkopf - II. [Kruskal symmetric solution - from Shepard, 1963, p. 39].....	33
3. A two-dimensional configuration of all International Morse Code signals based upon the same-or-different judgments obtained by Rothkopf - III. [Kruskal nonsymmetric solution].....	34
4. A two-dimensional configuration of all International Morse Code signals based upon the same-or-different judgments obtained by Rothkopf - IV. [Guttman-Lingoes (Smallest Space Analysis- II) row solution].....	35
5. First two dimensions of a three-dimensional analysis of Rothkopf's data. [Guttman-Lingoes (Smallest Space Analysis - II) row solution].....	37
6. A two-dimensional configuration for the 32 Morse-like rhythmic patterns - I. [Guttman-Lingoes (Smallest Space Analysis - II) row solution].....	72
7. A two-dimensional configuration for the 32 Morse-like rhythmic patterns - II. [Guttman-Lingoes (Smallest Space Analysis - II) row solution].....	74
8. A two-dimensional configuration for the 32 Morse-like rhythmic patterns - III. [Kruskal nonsymmetric solution].....	76
9. First and second dimensions of a three-dimensional configuration for the 32 Morse-like rhythmic patterns. [Guttman-Lingoes (Smallest Space Analysis - II) column solution].....	77
10. First and third dimensions of a three-dimensional configuration for the 32 Morse-like rhythmic patterns. [Guttman-Lingoes (Smallest Space Analysis - II) column solution].....	78
11. Second and third dimensions of a three-dimensional configuration for the 32 Morse-like rhythmic patterns - I. [Guttman-Lingoes (Smallest Space Analysis - II) column solution].....	80
12. Second and third dimensions of a three-dimensional configuration for the 32 Morse-like rhythmic patterns - II (signals beginning with .1. or -3- are omitted). [Guttman-Lingoes (Smallest Space Analysis - II) column solution].....	81

LIST OF FIGURES (continued)

Figure	Page
13. A two-dimensional configuration for the 32 Morse-like rhythmic patterns based on data from the first third of the experiment. [Kruskal nonsymmetric solution].....	82
14. A two-dimensional configuration for the 32 Morse-like rhythmic patterns based on data from the final third of the experiment. [Kruskal nonsymmetric solution].....	83